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Optimal insurance design under background risk with dependence

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ABSTRACT

In this paper, we revisit the problem of optimal insurance under a general criterion that preserves stop-loss order when the insured faces two mutually dependent risks: background risk and insurable risk. According to the local monotonicity of conditional survival function, we derive the optimal contract forms in different types of interval. Because the conditional survival function reflects the dependence between background risk and insurable risk, the dependence structure between the two risks plays a critical role in the insured's optimal insurance design. Furthermore, we obtain the optimal insurance forms explicitly under some special dependence structures. It is shown that deductible insurance is optimal and the Mossin's Theorem is still valid when background risk is stochastically increasing in insurable risk, which generalizes the corresponding results in Lu et al. (2012). Moreover, we show that an individual will purchase no insurance when the sum of the two risks is stochastically decreasing in insurable risk.
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1. Introduction

The problem of optimal insurance design has drawn significant interest in both research and practice since the seminal papers by Borch (1960) and Arrow (1963) and lots of models have been formulated and studied extensively. According to the sources of risk assumed in the model, the studies of optimal insurance can be classified into the following two categories: the models with one source of risk and the models with background risk.

The former are based on the assumption that there is only one source of risk which can be entirely insured; see, e.g., the survey paper Bernard (2013), Loubergé (2013), Gollier (2013) and recent literatures such as Carlier and Dana (2003), Karni (2008), Guerra and Centeno (2008), Guerra and Centeno (2010), Chi and Lin (2014), Cui et al. (2013), Gollier (2014), Cheung et al. (2015), Bernard et al. (2015), Golubin (2016), Li and Xu (2017), and the references therein. However, the assumption of one source of risk is a simplification because almost everyone faces risks that can be quantified but cannot be insured, for example, war, floods, market valuation of stocks, inflation and other general economic conditions. These uninsurable risks are generally called background risks.

Most studies have arrived at the same conclusion that the addition of an independent background risk does not alter the form of the optimal insurance derived under the assumption of one source of risk but does affect optimal level (see Briys and Viala (1995), Gollier (1996), Mahul (2000) and Lu et al. (2012)). However, as noted in Briys and Viala (1995), the classical results with one source of risk such as Raviv (1979), do not necessarily hold if there exists positive dependence between the insurable risk and the background risk. Hence, the presence of background risk will affect the optimal form and level of the insurance contract. Exploring optimal insurance design under stochastic background risk led to the second type of optimal insurance problem.

In the past thirty years, numerous papers have been dedicated to studying the optimal insurance policy in the presence of background risk. The studies on this issue can be divided into two categories according to whether the contractual forms are known. Some of the research attempt to determine the optimal level based on the assumption that contractual forms are restricted to specific types of insurance, such as deductible insurance or proportional insurance. For example, Doherty and Schlesinger (1983) have explored the choice of an optimal deductible in an insurance contract when initial wealth is random and show that Mossin's Theorem[1] holds if the correlation between initial wealth and insurable losses is negative or zero, but the principle may not be valid if the correlation is positive. In Jeleva (2000), optimal proportional insurance was analyzed in a nonprobabilized uncertainty framework,

[1] A well-known result in the literature is the Bernoulli principle, which postulates that full coverage is optimal if insurance premiums are actuarially fair. Mossin (1968) extended this result to show that less than full coverage is optimal when the insurance premium includes a positive, proportional loading. In this study, the aforementioned results are generally referred to as Mossin's Theorem.

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where the preferences are represented using the Choquet expected utility model. Luciano and Kast (2001) studied the effects of an uninsurable background risk on the demand for contractual forms of deductible insurance and proportional insurance under the Mean-VaR framework. To overcome the deficiencies of the classical correlation coefficient, Hong et al. (2011) re-examined Mossin's Theorem under random initial wealth by extending the correlation coefficient to more general dependence measures called positively (negatively) expectation dependent; see also Mayers and Smith (1983), Schlesinger (1997), Meyer and Meyer (1998), Guiso and Jappelli (1998) and references therein.

On the other hand, other studies aim to identify optimal contractual forms of insurance under some given constraints. Briys and Viala (1995) may be the first work on this issue. They extended Raviv's (1979) framework on the design of optimal policy to include the presence of an uninsurable background risk and revealed that the policy with a disappearing deductible is optimal under a specific positive dependence between the insurable risk and the background risk. Gollier (1996) studied the optimal additive insurance model[2] and showed that the optimal insurance policy may be a disappearing deductible if the uninsurable asset increases with the size of the insurable asset under the assumption that the policyholder is prudent. Mahul (2000) re-examined Raviv's (1979) result in the case that the insured faces both background risk and insurable risk. The author obtained a similar result to Gollier (1996) when an increase in the insurable loss makes riskier the policyholder's random initial wealth, according to any degree of stochastic dominance, and if the derivatives of his utility function alternate in sign. Vercammen (2001) revisited the problem of optimal insurance under the assumption that the background risk and the insurable risk are nonseparable with positive dependence. Contrary to the result of the disappearing deductible in Gollier (1996), Vercammen (2001) showed that the optimal contract is coinsurance above a deductible minimum when the insured is prudent. Dana and Scarsini (2007) examined qualitative properties of efficient insurance contracts in the presence of background risk under different assumptions on the stochastic dependence between the insurable and uninsurable risk. Lu et al. (2012) studied the problem of optimal insurance when the insurable risk and uninsurable background risk are positively dependent in the framework of expected utility. They showed that the deductible insurance is optimal and Mossin's Theorem still holds. Furthermore, the shifts of optimal deductible and expected utility by modification of the dependence structure and the marginal are analyzed. Huang et al. (2013) discusses the optimal insurance contract endogenously under the assumption that background risk depends on insurable loss. Based on some specific assumptions, the optimal insurance policies were derived under the expected utility and mean–variance frameworks and the results under different frameworks are compared. In a recent study by Chi (2015), the author investigated the optimal form of insurance under the mean–variance framework by imposing an incentive-compatible constraint on the coverage function. With the help of a constructive approach, Chi (2015) derived the optimal insurance form explicitly, which depends heavily on the conditional expectation function of the background risk with respect to the insurable risk.

For an insurance contract, the optimal form and quantity, from the insured's point of view, depend on his optimization criterion. Many optimization criteria such as expected-utility criterion, variance criterion, adjustment coefficient criterion, the criteria based on the distortion risk measure (including VaR, TVaR) etc., have been proposed for the derivation of the optimal (re)insurance. Nevertheless, as noted by Denuit and Vermandele (1998), most criteria often reduce to the comparison of certain characteristics of the retained risks and cannot been used to take into account all the characteristics of the underlying probability distribution.

During the past forty years, the problem of comparing risks lies at the heart of insurance business and the theory of stochastic order provides a powerful analysis tool in studying the optimal insurance. A more general criterion that preserves stop-order (convex order) has been adopted in some studies of optimal insurance. The criterion that preserves stop-loss order is first formally proposed by Van Heerwaarden et al. (1989). Gollier and Schlesinger (1996) proved Arrow's theorem on the optimality of deductibles under the optimization criterion of second-degree stochastic dominance (i.e. stop-loss order), without invoking the expected-utility hypothesis. Denuit and Vermandele (1998) derived new results about the optimal reinsurance coverage , when the optimality criterion consists in minimizing the retained risk with respect to the stop-loss order. In a recent study on the optimal reinsurance by Chi and Tan (2011), the criterion that preserves convex order has been adopted as optimization criterion. One of the advantages of the criterion that preserves stop-order (convex order) is that most of the probability characteristics of the retained risk can be considered simultaneously. In addition, many usual optimization criteria including expected-utility maximizing criterion, variance minimizing criterion, adjustment coefficient maximizing criterion, TVaR criterion, just name a few, are all special cases of it (see Van Heerwaarden et al. (1989), Denuit and Vermandele (1998), Dhaene et al. (2006), etc.), that is, the optimal insurance policy under the criterion that preserves stop-loss order will also be the most desirable with respect to these criteria.

When we study optimal insurance under background risk with dependence, the structure of dependence between the two risks needs to be considered. Some concepts of dependence between two random variables have been introduced. For example, Hong et al. (2011) re-examined Mossin's Theorem under random initial wealth by replacing the correlation coefficient with the structures of positive (negative) quadrant dependence, stochastic increasing (decreasing) dependence and positive (negative) expectation dependence. Among various notions of dependence, stochastic increasing (decreasing) dependence is interesting to model dependent risks and may be the most common one in the studying of optimal insurance (see Dana and Scarsini (2007), Cai and Wei (2012), Lu et al. (2012), etc.). So in this study, we will still describe the dependence between the insurable risk and background risk by using the concepts of stochastic increasing (decreasing) dependence.

In this paper, we extend the studies of Van Heerwaarden et al. (1989) and Gollier and Schlesinger (1996), and explore optimal insurance in the presence of background risk under a general criterion that preserves stop-loss order. According to the local monotonicity of the conditional survival function, we derive the optimal insurance form on different types of intervals by applying a constructive approach. Furthermore, we obtain the optimal insurance forms explicitly under some special dependence structures. It is shown that deductible insurance is optimal and Mossin's Theorem is still valid when background risk is stochastically increasing in insurable risk, which generalizes the corresponding results in Lu et al. (2012). Moreover, we show that an individual will purchase no insurance when the sum of the two risks is stochastically decreasing in insurable risk. In the case that $Y \uparrow_{st} X$, Lu et al. (2012) and Chi (2015) reached the same conclusion under the framework of expected utility and mean–variance respectively. However, under the framework with expected utility and the assumption that Y and X are positively dependent (not positive increasing dependent), Gollier (1996) and Dana and Scarsini (2007) showed that the optimal contract was a disappearing deductible , which would

[2] The Models of optimal insurance with background risk can also be classified into two groups. The first is concerned with the case where the risks are additive, and the second is related to the case where the risks are not additive.

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