J. Math. Anal. Appl. • • • (• • • •) • • •–• • •

Contents lists available at ScienceDirect

Journal of Mathematical Analysis and Applications

www.elsevier.com/locate/jmaa

# Robust optimal investment and reinsurance of an insurer under variance premium principle and default risk ☆

Zhongyang Sun [a], Xiaoxiao Zheng [a,*], Xin Zhang [b]

[a] School of Mathematics, Sun Yat-Sen University, Guangzhou 510275, China
[b] Department of Mathematics, Southeast University, Nanjing 210096, China

ARTICLE INFO

*Article history:*
Received 5 February 2016
Available online xxxx
Submitted by H.-M. Yin

*Keywords:*
Robust optimal control
Investment and reinsurance
Defaultable bond
CARA utility
Dynamic programming approach

ABSTRACT

This paper considers a robust optimal investment and reinsurance problem under model ambiguity and default risk for an insurer, who can trade in a saving account, a stock and a defaultable bond and aims to maximize the minimal expected CARA utility. The surplus process of the insurer is assumed to follow the Cramér–Lundberg model. In particular, both the insurance and reinsurance premium are assumed to be calculated via the variance premium principle. By using the dynamic programming approach, we study the pre-default case and post-default case respectively, then closed-form expressions for the optimal strategies and the corresponding value function are derived. Finally, numerical examples are given to illustrate our main results, and we discuss relevant economic insights obtained from these results.



## 1. Introduction

The study of investment and reinsurance has long been a hot issue for insurers both in academic and practical aspects. They are effective approaches for insurers to achieve financial objects and manage risk exposures. Nowadays, many scholars engage in this field and elegant results have been achieved. Based on different risk measurements: Bai and Guo [3], Chen et al. [13] and Schmidli [22] considered the optimal investment–reinsurance problem of an insurance company to minimize the ruin probability; Mean-variance portfolio selection problem proposed by Markowitz [21] was investigated by many literature, such as: Bäuerle [4], Celikyurt and Özekici [11], Zeng et al. [26] and so on; Browne [9] and Yang and Zhang [23] investigated optimal investment or reinsurance strategies when the object function is an exponential utility function.

☆ Supported by the National Natural Science Foundation of China (NSFC Grant Nos. 11371020, 11571189, 11601147).
* Corresponding author.
*E-mail addresses:* zysun_nk@126.com (Z. Sun), xxzh1022@163.com (X. Zheng), x.zhang.seu@gmail.com (X. Zhang).

http://dx.doi.org/10.1016/j.jmaa.2016.09.053
0022-247X/

Most of the literature mentioned above, however, are based on expected value premium principle due to its simplicity and popularity in practice. Against this principle, one can convincingly argue that two risks with the same mean will charge for the same amount of premium. But everyone agrees that the underlying danger may appear strongly different. Thus, one could therefore opt for the variance premium principle (with a safety loading proportional to the variance of the risk), which permits the insurer to take the fluctuations (variance) of claims into consideration when signing insurance contracts. Generally speaking, the expected value premium principle is commonly used in life insurance whose claim frequency and claim sizes are stable and smooth, while the variance premium principle is extensively used in property insurance. Hence in this paper, both the insurance and reinsurance premium payments are calculated by using the variance premium principle which is more suitable for our case.

Although so many notable scholars have investigated optimal investment–reinsurance strategies for insurers, two important issues are ignored by the existing literature: one is lack of considering for investment in defaultable assets, and the other one is model uncertainty. On one hand, with the rapid development of financial markets, high yield corporate bonds have become increasingly attractive to investors. Although default risk for them do exist, they are often sought after because of their relative high yields. In fact, portfolio optimization problems with defaultable securities have become an important field of study. Bielecki and Jang [5] discussed an optimal allocation problem among a defaultable bond, a saving account and a stock for the investor who aimed to maximize the CRRA utility function of the terminal wealth. Jin and Hou [16] also considered an optimal investment problem with default risk by employing the intensity based approach. In Bo et al. [6,7], the authors addressed optimal portfolio problems in a defaultable market under Log utility and HARA utility respectively. However, these literature mention above all failed to associate with insurance which plays an important role in modern economic life. Zhao et al. [28] first considered an optimal investment–reinsurance problem involving a defaultable security for an insurer under the mean-variance criterion in a jump-diffusion risk model. In their paper, time-consistent investment–reinsurance strategies and the corresponding optimal value functions are derived from a game theoretic perspective. Zhu et al. [30] extended the risk model of Zhao et al. [28] to a stochastic volatility case when the objective function of the insurer is an exponential utility function.

On the other hand, it is a common belief that there is no agreement on which model, or real world probability, should be used in practice. So model uncertainty exists widely in finance, especially in the field of asset pricing, consumption and portfolio selection. Chen and Epstein [12] formulates a continuous-time intertemporal version of multiple-priors utility, where aversion to ambiguity is admissible. In Cont [14], a quantitative framework for measuring model uncertainty was introduced in option pricing models. Fei [15] studied the optimal consumption and portfolio choice in a Merton-style model with ambiguity by adopting the recursive multiple-priors utility. As an extension, Maenhout [19] presented a new approach to the dynamic portfolio and consumption problem of an investor who worries about model uncertainty and seeks robust decisions. In accordance with max–min expected utility, a robust investor insures against some endogenous worst case. For other articles in this field, interested readers may refer to Cairns [10], Liu [18], Zheng et al. [29] and references therein.

Under model uncertainty, one possible way to ensure the effectiveness of decisions is to use the robust approach, where some alternative models closed to the estimated model are introduced and a robust optimal strategy is obtained. Recently, some papers on optimal investment–reinsurance strategies with ambiguity have come into being. Zhang and Siu [27] introduced a novel approach via a zero-sum stochastic differential game to solve the optimal investment–reinsurance problems of insurance companies when facing model uncertainty. But in their model, the stock is the only available asset for the insurer. Yi et al. [24] considered a robust optimal investment–reinsurance problem under Heston's SV model for an ambiguity-averse insurer, whose investment choice consists of one saving account and one stock. In Yi et al. [25], the authors also investigated investment–reinsurance strategies that are robust with respect to model uncertainty, and aimed to optimize their decisions in a mean-variance framework.

Please cite this article in press as: Z. Sun et al., Robust optimal investment and reinsurance of an insurer under variance premium principle and default risk, J. Math. Anal. Appl. (2016), http://dx.doi.org/10.1016/j.jmaa.2016.09.053