Stability Analysis of Discrete-Time Linear Time-Varying Switched Systems with Delays [⋆]

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Abstract: This paper addresses the stability issues of linear time-varying switched systems with time-varying delays. It is assumed that the delays are piecewise continuous, either bounded or unbounded, and that there exists a comparison system, formed by means of the lower and upper bounds of elements of the system matrices of the original system, being asymptotically or exponentially stable. With these assumptions, the stability conclusion of original system can be inferred from that of comparison system. An example is provided to illustrate the proposed theoretical result.

Keywords: Delays, positive systems, stability, switched systems, time-varying systems.

1 INTRODUCTION

Delays, especially time-varying ones, frequently arise in various engineering systems, including nuclear reactors from nuclear power plant, chemical engineering systems from chemical factory, and population dynamics of biological systems, to mention only a few. Generally speaking, delay is a source of system's instability and may lead to performance deterioration Mazenc (2015); Yang et al. (2009). Time-varying property is another factor to be handled in practical systems because the parameters of any system vary with time Bourlès and Marinescu (2011). In the context of linear systems, a system is time-varying means that the system matrices are functions in time t. The changing of characteristics may have important effects on system dynamics, for example, with changing parameters, a stable system may become unstable DaCunha (2007). Therefore, investigating properties, especially stability, of time-varying systems with delays has attracted much attention from the community of control theory and engineering Dehghani et al. (2012).

Though stability condition of linear time-invariant systems with constant delays has been established Curtain and Zwart (1995), many challenging stability problems remain open for linear time-varying systems with time-varying delays Gárate-García et al. (2011); Egorov and Mondié (2014). In the presence of time-varying delays, the Lyapunov-Krasovskii functional and Lyapunov-Razumikhin approaches are powerful and therefore are most frequently employed. There are two aspects that should not be ignored: 1). These methods require a Lyapunov-Krasovskii functional or Lyapunov-Razumikhin functional, which is hard to construct sometimes and is not easy to produce tight (low conservative) stability conditions Münz et al. (2009); Bresch-Pietri et al. (2014). 2). Additional relatively strong constraints are often imposed on delays, for example, in many papers delays are required to be differentiable, with a small upper bound of the derivative, or even constant Chen and Zheng (2010); Kim et al. (2008). These constraints limit the application of Lyapunov method. Actually, in the situation of positive system, many important results are derived based on non-Lyapunov approach Liu et al. (2010); Ngoc (2013). When the considered systems are additionally time-varying, as in the present paper, generally speaking, one cannot expect to propose a tight stability criterion by means of the Lyapunov methods.

Recall some interesting properties of positive systems with delays. A system is said to be positive if its trajectory remains in the positive quadrant whenever its initial function is nonnegative. It was revealed that the size of delay does not affect system's stability property Haddad and Chellaboina (2004). In other words, its stability is completed determined by system matrices. This property keeps true even in the case of delays being unbounded Liu and Dang (2011). Besides, time-varying positive systems also have good property Li and Lam (2011). For more other properties of positive systems, see Liu and Lam (2013); Shen and Zheng (2015) and the references therein. The stability of a class of time-invariant systems with bounded delays was studied in Feyzmahdavian et al. (2014), which

established an exponential stability condition for considered system by bounding the amplitude of the system trajectory. A class of linear time-invariant switched systems with unbounded delays is investigated in Sun (2012), the stability conclusion is claimed by showing the system trajectory tends to equilibrium.

Motivated by achievements in the field of positive systems, in the present paper we try to find other class of systems whose stability is not effected by their delays. Due to the fact that there is a number of challenges in the area of linear time-varying switched systems with time-varying delays, where the delays are only required to be piecewise continuous, we are urged to seek a subclass of such system for which their system matrices can be bounded by those of a stable positive switched system. Roughly speaking, it is our task to derive a stability conclusion of the involved switched system from that of the corresponding positive switched system.

Performing such a task is technically nontrivial. As mentioned above, it is advisable to discard the classical Lyapunov-Krasovskii or Lyapunov-Razumikhin functional method. The frequency approach does not work here, since it is applied for constant system only Partington (2004). The idea used here can be briefly summarized as follows: For given original time-varying switched systems with time-varying delays (A), construct a comparison positive switched system (B) whose system matrices are element-wise determined by the low and upper bounds of elements of original system matrices. If the comparison system is stable, then the following relation between systems (A) and (B) holds: The fact that the initial function of (A) is bounded by that of (B) implies the fact that the trajectory of (A) is bounded by that of (B), which indicates the stability of the original system.

The main contribution of the present paper lies in the following two aspects: 1). It is revealed that the trajectory of the original system can be bounded by that of comparison system provided that the amplitude of initial function of the original system is less than that of comparison system. 2). It is proved that the asymptotic or exponential stability of the original switched system can be inferred from that of the comparison system.

The rest of the paper is organized as follows: Preliminaries employed in the paper and the problems to be addressed are presented in Section 2. Main results are proposed in Section 3, an illustrative example is provided in Section 4, and summary conclusions are given in Section 5.

Notation: $|a|$ is the absolute value of a real number a. $\mathbf{0}$ stands for the n-dimensional zero vector. A^{T} is the transpose of matrix A. A is said to be a Metzler matrix if all its off diagonal entries are nonnegative. For given vector $\boldsymbol{\lambda}$, $\boldsymbol{\lambda} \succeq (\succ, \preceq, \prec)\mathbf{0}$ means that all elements of $\boldsymbol{\lambda}$ are nonnegative (positive, nonpositive, negative). $\mathbb{R}^{n \times m}$ denotes the set of all real matrices of $n \times m$-dimension and $\mathbb{R}^n = \mathbb{R}^{n \times 1}$. $\mathbb{R}_{0,+} = [0, +\infty), \mathbb{R}_+ = (0, +\infty)$. $\mathbb{N}_0$ stands for the set of nonnegative integers and $\mathbb{N} = \mathbb{N}_0 \setminus \{0\}$. $\underline{m} = \{1, \ldots, m\}$ for $m \in \mathbb{N}$. Given $\boldsymbol{x} = [x_1, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$, $\|\boldsymbol{x}\|_\infty = \max\{|x_1|, \ldots, |x_n|\}$ is the l_∞ norm $\boldsymbol{x}$, and for simplicity, is denoted by $\|\boldsymbol{x}\|$, $|\boldsymbol{x}| = [|x_1|, \ldots, |x_n|]^{\mathrm{T}}$. $\mathcal{C}([a, b], X)$ is the set of continuous functions from interval $[a, b]$ to X. For any continuous function $\boldsymbol{x}(s)$ on $[-d, a)$ with scalars $a > 0$, $d > 0$ and any $t \in [0, a)$, $\boldsymbol{x}_t$ denotes a continuous function on $[t - d, t]$ defined by $\boldsymbol{x}_t(\theta) = \boldsymbol{x}(t + \theta)$ for $-d \leq \theta \leq 0$; for any real number c, $c\boldsymbol{x}_t = c\boldsymbol{x}(t + \theta)$ for $-d \leq \theta \leq 0$; $\|\boldsymbol{x}_t\| = \sup_{t-d \leq s \leq t}\{\|\boldsymbol{x}(s)\|\}$. Throughout this paper, the dimensions of matrices and vectors will not be explicitly mentioned if clear from context.

2 PRELIMINARIES AND PROBLEM STATEMENTS

Consider the following system:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= A(t)\boldsymbol{x}(t) + B(t)\boldsymbol{x}(t - d(t)), \quad t \geq t_0 \\ \boldsymbol{x}(t) &= \boldsymbol{\varphi}(t), \quad t \in [t_0 - d, t_0] \end{aligned} \quad (1)$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state variable, $A(t) = [a_{ij}(t)] \in \mathbb{R}^{n \times n}, B(t) = [b_{ij}(t)] \in \mathbb{R}^{n \times n}$ are time-varying system matrices. Delay $d(t)$ is assumed to be piecewise continuous with respect to t and satisfies either of the next two conditions:

$$0 \leq d_1 \leq d(t), \ -d \leq t - d(t), \forall t \geq t_0, \lim_{t \to \infty}(t - d(t)) = \infty \quad (2)$$

$$0 \leq d_1 \leq d(t) \leq d, \quad \forall t \geq t_0 \quad (3)$$

with $d_1 \in \mathbb{R}_{0,+}, d \in \mathbb{R}_+$ being constant. $\boldsymbol{\varphi} : [t_0 - d, t_0] \to \mathbb{R}^n$ is the vector-valued initial function. Note that delays satisfying (2) is discussed in Sun (2012) and may be unbounded. Clearly, the constraint on delays is very mild. For simplicity, use $\dot{\boldsymbol{x}}(t_0)$ to denote the right-hand derivative of $\boldsymbol{x}(t)$ at t_0.

Now provide some definitions and lemmas for later use.

Definition 1. (Khalil (2002)). A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$, where a is any positive real number or ∞, and is said to belong to class $\mathcal{K}_\infty$ if it belongs to class $\mathcal{K}$ and $\alpha(r) \to \infty$ as $r \to \infty$. A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{KL}$ if, for each fixed s, the mapping $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.

Definition 2. (Mahmoud (2010)). Denote $\boldsymbol{x}(t; t_0, \boldsymbol{\varphi})$ the solution of (1) with starting time t_0 and initial function $\boldsymbol{\varphi}$ defined on interval $[t_0 - d, t_0]$. System (1) is uniformly exponentially stable if there exist scalars $\alpha \geq 1$ and $\gamma > 0$ such that $\|\boldsymbol{x}(t; t_0, \boldsymbol{\varphi})\| \leq \alpha \exp(-\gamma(t - t_0))\|\boldsymbol{\varphi}\|$, $\forall t_0 \geq 0, t \geq t_0, \boldsymbol{\varphi}(\cdot) \in \mathcal{C}([t_0 - d, t_0], \mathbb{R}^n)$. System (1) is uniformly asymptotically stable if there exists a class $\mathcal{KL}$ function β such that $\|\boldsymbol{x}(t; t_0, \boldsymbol{\varphi})\| \leq \beta(\|\boldsymbol{\varphi}\|, t - t_0), \forall t_0 \geq 0, t \geq t_0, \boldsymbol{\varphi}(\cdot) \in \mathcal{C}([t_0 - d, t_0], \mathbb{R}^n)$.

3 MAIN RESULTS

This section will discuss the stability issue of linear time-varying switched systems with time-varying delays described by

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= A_{\sigma(t)}(t)\boldsymbol{x}(t) + B_{\sigma(t)}(t)\boldsymbol{x}(t - d_{\sigma(t)}(t)), t \geq t_0 \\ \boldsymbol{x}(t) &= \boldsymbol{\varphi}(t), \quad t \in [t_0 - d, t_0] \end{aligned} \quad (4)$$

where the mapping $\sigma : [t_0, \infty) \to \underline{m}$ is a switching signal with m being the number of subsystems, which is piecewise constant and continuous from the right. In our context, the switching signal σ is with switching sequence $\{t_i\}_{i=0}^{\infty}$