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Sufficient stability conditions of fractional systems with perturbed differentiation orders

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Abstract: In this paper **sufficient stability conditions** are established for fractional systems with perturbed differentiation orders. It is an extension of the recently published paper Rapaić and Malti [2016] which allows henceforth increasing the highest differentiation order. The maximum allowable variation on all differentiation orders is computed so that the stability (respectively instability) is preserved when differentiation orders are perturbed away from the commensurate ones. The maximum allowable variations are compared in both cases: when the highest order is allowed to be increased and when it is not. The established conditions allow concluding on the stability of incommensurate fractional systems on the basis of Matignon's theorem and the additional sufficient condition.

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1. INTRODUCTION

One of the most used criterion for stability testing of fractional systems is Matignon [1998] theorem. It extends the Routh-Hurwitz criterion by testing whether system s^ν-poles, where ν is the commensurate order, are located in the sector $|\arg(s^\nu)| > \nu\frac{\pi}{2}$. However, when the fractional system is incommensurate, Matignon's theorem does no more apply and some other criteria were developed. Most of these criteria ultimately derive form the Cauchy's Argument Principle, either directly like Hwang and Cheng [2006], or indirectly, through modification of the Nyquist Theorem, like Trigeassou et al. [2009], Sabatier et al. [2013]. However, al of these methods are quite difficult to implement because they require computing all the s-poles in the complex right half plane, and carefully excluding the branch cut lying generally on the negative real axis.

Recently, sufficient conditions for testing stability of fractional incommensurate systems were established in Rapaić and Malti [2016]. They allow concluding on the stability (resp. instability) of incommensurate systems when a neighboring commensurate system is stable (resp. unstable). Moreover, a conservative upper bound on the magnitude of the allowable variations of the differentiation orders was determined, such that the perturbed system remains stable (resp. unstable). However, in the aforementioned paper, *the maximum differentiation order (system order) is not to be increased by the perturbation* (the highest order perturbation is non positive). **This limitation is raised in this paper.** Henceforth, new conditions are established allowing to have perturbations augmenting system order (any highest order perturbation). It is expected, in the latter case that the maximum allowable perturbation be reduced as compared to the former.

The paper is organized as follows. First a mathematical background is presented on fractional systems together with Matignon's stability theorem. Then, the main results are presented in Section 2. The ones established recently in Rapaić and Malti [2016] are first recalled in section 2.1 as Theorems 3 and 5. Next, the extension allowing to handle an increase in the highest differentiation order is presented in Section 2.2 as Theorems 6 and 7. Numerical examples of Section 3 show that the maximum allowable perturbation reduces when perturbations are allowed to increase the highest differentiation order as compared to the case when they do not.

1.1 *Mathematical background*

A symbolic representation of a fractional dynamic system governed by a fractional differential equation is given in a **transfer function** form:

$$\frac{T(s,\beta)}{F(s,\alpha)} = \frac{\sum_{j=0}^{m} b_j s^{\beta_j}}{1+\sum_{i=1}^{n} a_i s^{\alpha_i}}, \tag{1}$$

where $a_i \in \mathbb{R}^* \; \forall i \in \{1,2,\ldots n\}$, $b_j \in \mathbb{R}^* \; \forall j \in \{0,1,\ldots m\}$, and where $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\beta = (\beta_0, \beta_1, \ldots, \beta_m)$ are vectors of ordered differentiation orders:

$$\begin{aligned} 0 < \alpha_1 < \alpha_2 < \ldots < \alpha_n\,, \\ 0 \le \beta_0 < \beta_1 < \ldots < \beta_m\,. \end{aligned} \tag{2}$$

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A common assumption is that the transfer function is strictly proper, i.e. that high-frequency gain is zero, which implies $\beta_m < \alpha_n$.

If the transfer function (1) is commensurate of order ν, then it can be rewritten as a ratio of two polynomials in s^ν:

$$\frac{T'(s^\nu)}{F'(s^\nu)} = \frac{\sum_{j=0}^{m} b_j s^{j'\nu}}{1+\sum_{i=1}^{n} a_i s^{i'\nu}}, \tag{3}$$

where $j' = \frac{\beta_j}{\nu}$ and $i' = \frac{\alpha_i}{\nu}$ are integers.

The stability addressed in this paper is the Bounded Input Bounded Output (BIBO) stability. The system described by $\frac{T}{F}$ in (1) is $\mathscr{L}_p$-stable, $1 \leq p \leq \infty$, if and only if

$$\sup_{u\in\mathscr{L}_p, u\neq 0} \frac{\|g \star u\|_p}{\|u\|_p} < \infty, \tag{4}$$

where $\star$ stands for the convolution product, g the inverse Laplace transform (impulse response, or kernel) of $\frac{T}{F}$ (or $\frac{T'}{F'}$) and $u(t)$ is the system input. The **Bounded-Input-Bounded-Output (BIBO)** stability is defined as the $\mathscr{L}_\infty$-stability.

In the case of fractional systems Bonnet and Partington [2000] extended the well-known result regarding stability of rational systems.

Theorem *1.* (Bonnet and Partington [2000]). Let $\frac{T}{F}$ be defined as in (1) with $\alpha_n \geq \beta_m$. Then $\frac{T}{F}$ is BIBO stable if and only if $\frac{T}{F}$ has no pole in the closed right-half plane $\{s : \mathscr{R}e(s) \geq 0\}$ (in particular no poles of fractional order as $s = 0$).

This theorem, conjectured in Skaar et al. [1988], Oustaloup [1995], Matignon [1998], will be used later in this paper. Further, Matignon [1998] established a very useful result regarding stability of commensurate fractional systems.

Theorem *2.* (Matignon [1998] extended). A commensurate transfer function $\frac{T'}{F'}$ with a commensurate order ν, as in (3), with T' and F' two coprime polynomials, is stable if and only if

$$0 < \nu < 2 \tag{5}$$

and

$$\forall s \in \mathbb{C} \text{ such that } F'(s) = 0,\ |\arg(s)| > \nu\frac{\pi}{2}. \tag{6}$$

Matignon initially proved this theorem for $0 < \nu < 1$. The proof was extended in multiple references to the interval $(1, 2)$, see e.g. Malti et al. [2011].

2. MAIN RESULTS

Section 2.1 recalls the results established in Rapaić and Malti [2016] when the highest differentiation order is not increased. Then, the extension proposed in this paper, allowing to handle variations in the highest differentiation order, is presented in Subsection 2.2 as Theorems 6 and 7.

2.1 Perturbations not increasing α_n

According to Theorem 1, if numerator and denominator of $\frac{T}{F}$ defined in (1) contain no common factor, the stability is determined only by poles position. It is therefore natural to formulate the results in terms of the characteristic function F.

Theorem *3.* Let $\alpha = (\alpha_1, \ldots, \alpha_n) \in \mathbb{R}^n$, and let

$$F(s,\alpha) = 1 + \sum_{i=1}^{n} a_i s^{\alpha_i}\ , \tag{7}$$

where $0 < \alpha_1 < \ldots < \alpha_n$. Let further $\epsilon = (\epsilon_1, \ldots, \epsilon_n) \in \mathbb{R}^n$ be such that

$$0 < \alpha_1 + \epsilon_1 < \ldots < \alpha_n + \epsilon_n \leq \alpha_n\ . \tag{8}$$

If

$$|F(j\omega,\alpha)| > \left|\sum_{i=1}^{n} a_i\epsilon_i \operatorname{Ln}(j\omega) \int_0^1 (j\omega)^{\alpha_i+t\epsilon_i} dt\right| \tag{9}$$

for all $\omega \in (0, \infty)$, then $F(s, \alpha)$ and $F(s, \alpha + \epsilon)$ have the same number of zeros in the closed complex right half plane. $\operatorname{Ln}(s)$ is the principal value of the complex logarithm.

Proof: The real and the imaginary parts of $F(s, \alpha)$ independently satisfy conditions of Theorem A, where one needs to formally substitute $x = 0$ and $h = \epsilon$. By adding the two expressions together (and multiplying the one regarding the imaginary part with j first) one obtains

$$\begin{aligned} F(s,\alpha+\epsilon) &= F(s,\alpha) + \sum_{i=1}^{n} \int_0^1 \left.\frac{\partial F(s,\alpha+\epsilon)}{\partial\epsilon_i}\right|_{\epsilon\leftarrow t\epsilon} \epsilon_i \\ &= F(s,\alpha) + \sum_{i=1}^{n} a_i \operatorname{Ln}(s)\epsilon_i \int_0^1 s^{\alpha_i+t\epsilon_i} dt\ . \end{aligned}$$

Now, by applying Rouché Theorem one sees that if

$$|F(s,\alpha)| > \left|\sum_{i=1}^{n} a_i\epsilon_i \operatorname{Ln}(s) \int_0^1 s^{\alpha_i+t\epsilon_i} dt\right| \tag{10}$$

on the contour C depicted in Fig. 1 with $\varepsilon \to 0$ and $R \to \infty$, then $F(s, \alpha + \epsilon)$ and $F(s, \alpha)$ have the same number of zeros within the closed right-half plane. By symmetry of the contour and of the Laplace transform, only the upper half of the contour should be checked.

i) On the imaginary axis, (10) reduces to the condition (9) stated in the formulation of the Theorem.

ii) On the quarter-circle with radius tending to zero, the Laplace variable can be substituted by $s = \varepsilon e^{j\varphi}$, with $\varphi \in [0, \pi/2]$ and $\varepsilon \to 0$.

$$1 > \lim_{\varepsilon\to 0} \left|\sum_{i=1}^{n} a_i\epsilon_i \operatorname{Ln}(\varepsilon e^{j\varphi}) \int_0^1 (\varepsilon e^{j\varphi})^{\alpha_i+t\epsilon_i} dt\right| \tag{11}$$

Since the right-hand side vanishes with ε, the condition is satisfied.

iii) On the quarter-circle with infinitely enlarging radius, the Laplace variable can be substituted by $s = Re^{j\varphi}$, with $\varphi \in [0, \pi/2]$ and $R \to \infty$. The upper bound on the right hand side of (10) is obtained in a straightforward manner,

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