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Stability Analysis of Control Systems subject to Delay-Difference Feedback

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Abstract: In engineering practice, delay-difference is often used to approximate the derivatives of output signals for feedback control, leading to a closed-loop system with delay both in the states and in the system's coefficients. In this context, our objective is to find all the delay values contained in some interval that guarantee the exponential stability of the closed-loop system subject to the delay-difference approximation. A method for stability analysis of systems with delay-dependent coefficients developed in our previous work is further extended and applied to tackle the particular form of systems considered in this paper. The proposed stability analysis procedure is illustrated through the design of a mobile-robot path-following controller.

Keywords: Time-delay Systems, Stability, Delay-dependent Coefficients, τ-decomposition.

1. INTRODUCTION

Feedback control design using only the output is very common in engineering practice due to the difficulty in measuring all the state variables. Quite common in practice, a static feedback of the output is not sufficient for stabilizing the system or to ensure satisfactory performance. Several strategies are available to deal with this problem. For instance, one may design an observer to reconstruct the entire state '$x(t)$' based on the output '$y(t)$'. Another strategy commonly taken in practice is to use the time derivative of the output, which leads to simpler controllers in comparison with observer-based control design. Since the derivative of the output usually can not be measured directly, it is often approximated by the following finite difference:

$$\dot{y}(t) \approx \frac{y(t) - y(t-\tau)}{\tau}, \tag{1}$$

where τ represents some positive delay value assumed sufficiently small. As a consequence of using delay-difference approximation (1) in the feedback, the closed-loop system becomes a *delay system* with *delay-dependent parameters*. We will not restrict our analysis to small delay values corresponding to the degrading effect specific to PD control schemes.

The idea of using delay for stabilization is not new. For instance, a multiple delay framework is developed in Niculescu & Michiels (2004) for stabilizing a chain of integrators, while in Yamanaka & Shimemura (1993) multiple delays are used for analysing some internal model control scheme. Bounded control for global stabilization has also been addressed in Mazenc, Mondie & Niculescu (2003), where a single delay is used. Our research differs from the previous ones in that we fix the other parameter of the controller while looking for the range of the delay parameter which guarantees that the closed-loop system is exponentially stable with some pre-specified decay rate α.

Systems with delay-dependent coefficients can be found in various scientific disciplines such as biological systems, e.g. Fabien (2005) and physical systems, see, for instance, Wilmot-Smith et al. (2006)). A large amount of research effort has been dedicated to stability analysis of delay systems. Readers may refer to Gu, Kharitonov & Chen (2003); Niculescu (2001); Michiels & Niculescu (2014) for comprehensive discussion of the related results. However, research on systems with delay-dependent coefficients is not common in the literature. In Beretta & Kuang (2002) an effective method is proposed for analysing stability of such systems with a single delay. Gu et al. (2016) relaxed some of their restrictive conditions and extended their approach for more general delay systems. Given a delay interval of interest denoted as $\mathscr{I}$, the method presented in Gu et al. (2016) can be used to find all the sub-intervals in $\mathscr{I}$ that guarantees asymptotic stability of the system. It can be considered as a generalization of the classical τ-decomposition approach, see, for instance, Michiels & Niculescu (2014); Lee & Hsu (1969).

The remaining part of this paper is organized as follows. We first specify the form of control law considered in this paper and the characteristic equation of the linearized closed-loop system resulting from the control design. Then it is shown that by shifting the variable in the characteristic equation, the condition for exponential stability with decay rate α is equivalent to a condition for just asymptotic stability. Next, we will make some further extension of the method developed in Gu et al. (2016) so that it can be used for the stability analysis of the control system considered in this paper. Finally the proposed design

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and analysis procedure is applied to a path-following control problem for illustration. The notations are standard.

2. MOTIVATING EXAMPLE

Consider a standard robot path following problem discussed in Lapierre & Jouvencel (2008) with some simplification. As illustrated in Fig.1, a unicycle travelling at a constant speed V follows a straight path. The robot is assumed to be non-holonomic, so the direction of its translational velocity is always along its heading direction. The control input u is the derivative of its yaw rate, which reflects the yaw moment applied to the robot. It is easy to see the linearized dynamics of the system is described by

$$\begin{cases} \dot{e} = V\theta \\ \dot{\theta} = \omega \\ \dot{\omega} = u \\ y = (\omega \;\; e)^T, \end{cases} \tag{2}$$

where e stands for the lateral tracking error, θ is the heading angle of the robot and ω is the yaw rate. The signal y is the output vector measured by the on-board sensors.

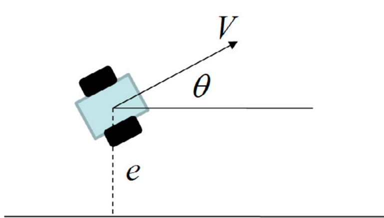

Fig. 1. Illustration of the robot path-following problem

There exist real numbers k_0, k_1, k_2 such that by choosing

$$u = -k_0\omega - k_1 e - k_2 V\theta,$$

the system can be stabilized. In practice, θ is not convenient to measure, therefore we do not include it in the output y. Noticing $\theta = V^{-1}\dot{e}$ and $\dot{e} \approx \frac{e(t)-e(t-\tau)}{\tau}$, we choose instead the following control law that uses delayed signal:

$$u = -(k_0 \;\; k_1)y - (0 \;\; k_2)\frac{y(t)-y(t-\tau)}{\tau} \tag{3}$$

with some appropriate values of the coefficients. By using some continuity type arguments, it can be shown that if system (2) can be stabilized by the following control law for some fixed real gains k_0, k_1, k_2:

$$u = -(k_0 \;\; k_1)y - (0 \;\; k_2)\dot{y}, \tag{4}$$

then it can also be stabilized by (3) for sufficiently small delay. However, if the delay value is too small, the noise contained in the measurement of y will be greatly amplified and injected into the closed-loop system and thus severely deteriorates the performance. On the other hand, a too large value of τ may cause slow convergence, strong oscillation, or even instability. Therefore for practical consideration it is useful to find a set of delay value for (3) such that the closed-loop system is exponentially stable with some guaranteed decay rate and then one can choose an appropriate delay value in this set.

3. PROBLEM STATEMENT

3.1 The PD Control Scheme

Consider a linear system of the form

$$\dot{x} = Ax + Bu, \tag{5}$$

where $x \in \mathbb{R}^n$ is the system state and $u \in \mathbb{R}$ is the control input. Let $y = Cx$ be the measured output vector. We assume the followings:

Assumption 1. There exist gain matrices K_1, K_2 such that the feedback law

$$u(t) = K_0 y(t) + K_1\dot{y}(t) \tag{6}$$

stabilizes the origin of (5)-(6). Furthermore, the matrix K_1 satisfies $K_1CB = 0$.

The condition $K_1CB = 0$ is imposed to avoid control signal $u(t)$ appearing also on the right-hand side of (6), causing an algebraic loop. Consequently, the system (5)-(6) is guaranteed to be well-posed.

The signal $\dot{y}$ usually can not be obtained directly from sensor measurement. In this case, delay-difference of $y(t)$ can be used to approximate $\dot{y}(t)$ and (6) thus becomes

$$u(t) = K_0 y(t) + K_1\frac{y(t)-y(t-\tau)}{\tau}, \tag{7}$$

where $\tau > 0$ is a constant number. For the closed-loop system consisting of (5) and (7), if the system trajectory x_t converges to zero exponentially fast with decay rate α, then we say the system is α-stable. Otherwise the system is α-unstable. For some fixed K_0, K_1 and a given delay interval $(0, \tau^u)$ as well as some non-negative decay rate α, we are interested in finding all the subintervals contained in $(0, \tau^u)$ such that the closed-loop system is α-stable for all τ in these subintervals.

3.2 Discussion

Although the feedback in (6) only involves the term $y(t)$ and $\dot{y}(t)$, however in practice a variety of stabilization problems can be converted into this form by first introducing a set of auxiliary state variables, which is then incorporated into the feedback. One thus constructs a control law based on dynamic output feedback. For instance, the classical PID controller can be constructed by first introducing an extra state variable σ satisfying:

$$\dot{\sigma} = y.$$

Now, we can define $u = K_I\sigma + v$ and choose a PD control law for the new control input v as $v = K_P y + K_D\dot{y}$ to construct the PID control.

As noted in Niculescu & Michiels (2004), when the open-loop system possesses more than a pair of imaginary roots, then it is necessary to introduce multiple delays in order to stabilize. We will leave this issue to our future work and restrict ourselves to control system with a single delay in this paper.

4. STABILITY ANALYSIS

4.1 Characteristic Equation and Stability

Let $G(\lambda) = 0$ be the characteristic equation of the open-loop system (5) with $u \equiv 0$, then $G(\lambda)$ is a polynomial in λ of degree n. There exist polynomials G_{u1}, G_{u2} such that the characteristic equation of the control system consisting of (5)-(6) takes the form

$$G(\lambda) + G_{u1}(\lambda) + G_{u2}(\lambda)\cdot\lambda = 0, \tag{8}$$

where $G_{u1}(\lambda)$ and $G_{u2}(\lambda)\lambda$ are generated by the terms $K_0 y$ and $K_1\dot{y}$ in the control feedback (6), respectively. Due to Condition