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PARTIAL STABILITY ANALYSIS OF SOME CLASSES OF NONLINEAR SYSTEMS*

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Abstract A nonlinear differential equation system with nonlinearities of a sector type is studied. Using the Lyapunov direct method and the comparison method, conditions are derived under which the zero solution of the system is stable with respect to all variables and asymptotically stable with respect to a part of variables. Moreover, the impact of nonstationary perturbations with zero mean values on the stability of the zero solution is investigated. In addition, the corresponding time-delay system is considered for which delay-independent partial asymptotic stability conditions are found. Three examples are presented to demonstrate effectiveness of the obtained results.

Key words Nonlinear systems; partial asymptotic stability; Lyapunov function; sector nonlinearities; time-delay

2010 MR Subject Classification 34D20; 34K20; 93D30

1 Introduction

The partial stability analysis is fundamental and challenging research problem due to its applications in various branches such as mechanics, electrodynamics, population dynamics, etc.; see, for example, [1–6]. It should be noted that the problem is especially important for satellites attitude control [7–9]. This problem naturally arises in the cases where only a part of variables characterizing the dynamics of the considered system is of interest or where only stability with respect to a part of components of the state vector is in fact possible.

The basic approaches to the investigation of stability with respect to a part of variables were developed by V. V. Rumyantsev [2, 10]. It is worth mentioning that he has proposed a counterpart of the Lyapunov functions method for solving issues of partial stability. The Rumyantsev approaches have gotten deep and extensive development, and many interesting

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and important results were derived with applications to numerous practical problems; see [3–5, 11–14] and the references cited therein.

Nevertheless, during the past few years, we have seen an increasing interest to the problem of partial stability [5, 6, 15–18]. In particular, it should be noted that methods of partial stability analysis is an effective tool for studying the consensus problem for multi-agent systems [19–22].

One of the first results on the partial stability is well-known Lyapunov–Malkin Theorem [23]. In this theorem, conditions of asymptotic stability with respect to a part of variables were found in the critical case of several zero eigenvalues of the matrix of a linear approximation system.

Later, some generalizations of this theorem providing partial asymptotic stability conditions by linear approximation were obtained by various authors; see [2, 3, 5] and the references therein. Moreover, in [24], the Lyapunov–Malkin Theorem was extended to the case of an essentially nonlinear homogeneous system of the first approximation.

The goal of this article is further development and extension of these results. We consider a complex system describing interaction of two subsystems. It is assumed that the first subsystem is a nonlinear Persidskii-type system [25] with nonlinearities satisfying sector conditions, and the zero solution of the subsystem is asymptotically stable, whereas the second subsystem admits stable zero solution. Using the Lyapunov direct method and the comparison method, conditions are derived under which the zero solution of the complex system is stable with respect to all variables and asymptotically stable with respect to a part of variables.

Next, we study an impact of nonstationary perturbations with zero mean values on the considered system. We will show that if the first subsystem is essentially nonlinear, then perturbations do not disturb stability even in the case where their orders coincide with those of the right-hand sides of unperturbed equations.

Finally, the corresponding time-delay system is investigated for which, with the aid of the Lyapunov direct method and the Razumikhin approach, delay-independent conditions of stability with respect to all variables and asymptotic stability with respect to a part of variables are obtained.

2 Preliminaries

In this section, we present some notations and definitions used in this article.

Throughout, $\mathbb{R}$ stands for the field of real numbers, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\|\cdot\|$ is the Euclidean norm of a vector, and $\mathbb{R}^{n\times l}$ is the space of $n\times l$ matrices with real entries.

Let $\mathrm{diag}\{\lambda_1,\cdots,\lambda_n\}$ be the diagonal matrix with the elements $\lambda_1,\cdots,\lambda_n$. We write P^T for the transpose of a matrix P. For a symmetric matrix A, the notation $A>0$ ($A<0$) means that the matrix A is positive (negative) definite.

For a given number $h>0$, let $C([-h,0],\mathbb{R}^n)$ be the space of continuous functions $\varphi(\theta)$: $[-h,0]\to\mathbb{R}^n$ with the uniform (supremum) norm $\|\varphi\|_h=\sup\limits_{\theta\in[-h,0]}\|\varphi(\theta)\|$.

Definition 2.1 ([25]) A matrix $C\in\mathbb{R}^{n\times n}$ is called diagonally stable if there exists a matrix $\Lambda=\mathrm{diag}\{\lambda_1,\cdots,\lambda_n\}>0$ such that $\Lambda C+C^T\Lambda<0$.