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Financial market models with Lévy processes and time-varying volatility ☆

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Abstract

Asset management and pricing models require the proper modeling of the return distribution of financial assets. While the return distribution used in the traditional theories of asset pricing and portfolio selection is the normal distribution, numerous studies that have investigated the empirical behavior of asset returns in financial markets throughout the world reject the hypothesis that asset return distributions are normally distribution. Alternative models for describing return distributions have been proposed since the 1960s, with the strongest empirical and theoretical support being provided for the family of stable distributions (with the normal distribution being a special case of this distribution). Since the turn of the century, specific forms of the stable distribution have been proposed and tested that better fit the observed behavior of historical return distributions. More specifically, subclasses of the tempered stable distribution have been proposed. In this paper, we propose one such subclass of the tempered stable distribution which we refer to as the "KR distribution". We empirically test this distribution as well as two other recently proposed subclasses of the tempered stable distribution: the Carr–Geman–Madan–Yor (CGMY) distribution and the modified tempered stable (MTS) distribution. The advantage of the KR distribution over the other two distributions is that it has more flexible tail parameters. For these three subclasses of the tempered stable distribution, which are infinitely divisible and have exponential moments for some neighborhood of zero, we generate the exponential Lévy market models induced from them. We then construct a new GARCH model with the infinitely divisible distributed innovation and three subclasses of that GARCH model that incorporates three observed properties of asset returns: volatility clustering, fat tails, and skewness. We formulate the algorithm to find the risk-neutral return processes for those GARCH models using the "change of measure" for the tempered stable distributions. To compare the performance of those exponential Lévy models and the GARCH models, we report the results of the parameters estimated for the S&P 500 index and investigate the out-of-sample forecasting performance for those GARCH models for the S&P 500 option prices.

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1. Introduction

Since Mandelbrot (1963) introduced the Lévy stable (or α-stable) distribution to model the empirical distribution of asset prices, the α-stable distribution became the most popular alternative to the normal distribution, the latter distribution being rejected by numerous empirical studies that have found financial return series to be heavy-tailed and possibly skewed. Rachev and Mitnik (2000) and Rachev et al. (2005) have developed financial models with α-stable distributions and applied them to market and credit risk management, option pricing, and portfolio selection as will as discussing the major attacks on the α-stable models. A fair conclusion of the literature is that while the empirical evidence does not support the normal distribution, it is also not consistent with an α-stable distribution. The distribution of returns for assets has heavier tails relative to the normal distribution and thinner tails than the α-stable distribution. Partly in response to those empirical inconsistencies, various alternatives to the α-stable distribution were proposed in the literature. The "classical tempered stable" (CTS) distribution (Koponen (1995); Boyarchenko and Levendorskiĭ (2000) and Carr et al. (2002)) and the "modified tempered stable" (MTS) distribution (Kim et al. (2006)) are two examples; an extension of the CTS distribution named the "KR" distribution (Kim et al. (in press)) is another. These distributions, sometimes called the tempered stable distributions, have not only heavier tails than the normal distribution and thinner than the α-stable distribution, but also have finite moments for all orders.

The tempered stable distributions are used for constructing the exponential Lévy model. If the driving process is the CTS process, then the exponential Lévy model is called the CGMY model, and if the driving process is the MTS process or KR process, then we refer to the exponential Lévy models as the MTS model or the KR model, respectively.

The main problem with the exponential Lévy models is that they generate an incomplete market; that is, the equivalent martingale measure (EMM) of a given market measure is not unique in general. For this reason, we need a method to select one reasonable EMM in the incomplete market generated by an exponential Lévy model. One classical method in selecting an EMM is the Esscher transform presented by Gerber and Shiu (1994, 1996); another reasonable method is finding the "minimal entropy martingale measure" presented by Fujiwara and Miyahara (2003). While these methods are mathematically elegant and have a financial interpretation within the context of a utility maximization problem, empirically the model prices obtained from the EMM have not matched the market prices observed for options. The other method for handling the problem is to estimate the risk-neutral measure by using current option price data independent of the historical underlying distribution. This method can fit model prices to market prices directly, but it has a problem: the historical market measure and the risk-neutral measure need not to be equivalent and it conflicts with the no-arbitrage property for option prices. To overcome these drawbacks, one must estimate the market measure and the risk-neutral measure simultaneously, and preserve the equivalent property between two measures. One method for doing so is "the least-squares calibration with a prior" proffered by Cont and Tankov (2004). Basically, their method finds an EMM of the market measure that minimizes the least squares error of the model option prices relative to the market option prices.

In spite of the skewness and the fat-tail property of the driving process, the exponential Lévy model has been rejected by empirical evidence (e.g., the finding that there is volatility clustering). The Markov property of the exponential Lévy model is one reason for the rejection. GARCH option pricing models have been developed to price options under the assumption of a non-Markovian property, more precisely, the assumption of volatility clustering. GARCH models of Duan (1995) and Heston and Nandi (2000) are important works on the non-Markovian structure of asset returns with the normal innovation process, but the normal innovation process disregards the empirical innovation process of asset returns. Duan et al. (2004) enhanced the classical GARCH model by adding jumps to the innovation processes. Subsequently, Menn and Rachev (2005a,b) introduced an enhanced GARCH model with innovations which follow the smoothly truncated stable (STS) distribution.

In this paper, we present market models based on the tempered stable distributions and provide empirical tests of these distributions. First, we consider the CGMY, MTS, and KR models. Then we find their EMM using the method of least-squares calibration with a prior and verify empirically the advantages of the KR model. We can find the parameters of the EMM such that the least squares error of the KR model prices are less than the error of the CGMY and MTS model prices. The change of measure between two KR processes has more freedom than that of the CGMY and MTS, and this freedom provides some empirical benefit which will be discussed.

We then construct a new GARCH model that combines the volatility clustering property of Duan's GARCH model and the skewness and fat-tail property of the infinitely divisible distribution which induces the Lévy process. This combination approach was first attempted by Menn and Rachev (2005a,b) using the α-stable distribution. In this paper, we improve and extend their approach. We consider the GARCH model and apply infinitely divisible distribu-