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Time variation of higher moments in a financial market with heterogeneous agents: An analytical approach

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Abstract

A growing body of recent literature allows for heterogenous trading strategies and limited rationality of agents in behavioral models of financial markets. More and more, this literature has been concerned with the explanation of some of the stylized facts of financial markets. It now seems that some previously mysterious time series characteristics like fat tails of returns and temporal dependence of volatility can be observed in many of these models as macroscopic patterns resulting from the interaction among different groups of speculative traders. However, most of the available evidence stems from simulation studies of relatively complicated models which do not allow for analytical solutions. In this paper, this line of research is supplemented by analytical solutions of a simple variant of the seminal herding model introduced by Kirman [1993, Ants, rationality, and recruitment. Quarterly Journal of Economics 108, 137–156]. Embedding the herding framework into a simple equilibrium asset pricing model, we are able to derive closed-form solutions for the time-variation of higher moments as well as related quantities of interest enabling us to spell out under what circumstances the model gives rise to realistic behavior of the resulting time series.

JEL classification: G12; C61; D84

Keywords: Noise traders; Speculation; Heteroskedasticity; Local and non-local interactions

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1. Introduction

Until very recently, theoretical research in finance has largely ignored some of the really universal stylized facts of practically all available financial data. In fact, a glance at frequently used textbooks like the ones by O'Hara (1995) and Barucci (2003) shows that even their glossaries lack entries for some of the prevalent technical terms of the empirical finance literature. For example, while many developments in empirical finance are essentially motivated by the observation of non-Gaussian returns distributions with their 'fat tails' and temporal dependence of second moments leading to 'volatility clustering', these notions have been almost entirely absent from the theoretical literature.[1] While these phenomena have spurred the development of such seminal innovations like GARCH type and stochastic volatility models in empirical finance, their behavioral origins have apparently remained an almost unaccessible puzzle for a long time. Two reasons might be responsible for this neglect: first, the above features characterize the behavior of financial time series as a whole, while the interest in economic theory has typically been to spell out the effect of a change of one (endogenous) economic variable on other, exogenous variables. Even when allowing for an ensemble of traders, such a comparative statics approach is not appropriate for explaining universal conditional and unconditional stochastic properties. Second, the prevalent efficient market paradigm did, in fact, provide a very simple implicit answer to the question of the origin of *all* stylized facts of returns: since, in this framework, prices would reflect forthcoming news in an unbiased and immediate manner, any property of the returns distribution would simply reflect a similar feature of the distribution of new information items. As a corollary of the efficient market hypothesis, the 'news arrival process' would, therefore, have to come along with fat tails and clustering of important news. Unfortunately, this corollary can hardly be subjected to econometric scrutiny. On the other hand, enough evidence had been collected against the universal validity of the efficient market paradigm to motivate alternative, behavioral approaches which then mushroomed over the nineties.

First analyses of complex data-generating mechanisms based on interacting agents can be found in Kirman (1991, 1993) and DeGrauwe et al. (1993). While they did not focus then on the above stylized facts (not broadly acknowledged at that time among theoretical researchers), they both already showed that their models could mimic the random walk nature of asset prices and exchange rates although their data-generating processes were clearly different from a true random walk. Notably, both studies also investigated what might be described as secondary stylized facts: they applied certain popular econometric analyses to their simulated data and found similar behavior as with empirical records providing a possible explanation of, e.g. the forward premium puzzle of foreign exchange markets.

Evidence for volatility clustering as an emergent phenomenon of a multi-agent model appeared first in Grannan and Swindle (1994). While a large body of

[1]Perfectly the same holds for theoretical work on exchange rate determination where the same stylized facts have also remained unnoticed for a long time.