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# Gather-and-broadcast frequency control in power systems ☆

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ABSTRACT

We propose a novel frequency control approach in between centralized and distributed architectures, that is a continuous-time feedback control version of the dual decomposition optimization method. Specifically, a convex combination of the frequency measurements is centrally aggregated, followed by an integral control and a broadcast signal, which is then optimally allocated at local generation units. We show that our *gather-and-broadcast* control architecture comprises many previously proposed strategies as special cases. We prove local asymptotic stability of the closed-loop equilibria of the considered power system model, which is a nonlinear differential–algebraic system that includes traditional generators, frequency-responsive devices, as well as passive loads, where the sources are already equipped with primary droop control. Our feedback control is designed such that the closed-loop equilibria of the power system solve the optimal economic dispatch problem.



## 1. Introduction

The quintessential task of power system operation is to match electrical load and generation. The power balance in an AC power network can be directly accessed via the system frequency, making frequency regulation the fundamental mechanism to ensure the load-generation balance. This task is subject to operational constraints, system stability, and economic interests, and it is traditionally accomplished by adjusting generation in a hierarchical structure consisting of three layers: primary droop control, secondary automatic generation control (AGC), and tertiary control (economic dispatch). These layers range from fast to slow timescales, and from decentralized to centralized control architectures (Machowski, Bialek, & Bumby, 2008; Wood & Wollenberg, 1996).

With the increasing integration of variable renewable sources, such as wind and solar power, low-inertia power electronic generation, larger peak loads, such as electric vehicles, and liberalized reserve markets on increasingly slower times (and their accompanying deterministic frequency errors), power grids are subject to larger and faster fluctuations (Milligan et al., 2015). In such a distributed generation environment, frequency control requires more fast-ramping generators to act as spinning reserves nowadays mostly provided by gas-driven generation, which is expensive, inefficient, and the resulting emissions defeat the purpose of renewables (Leonhard & Muller, 2002). As a partial remedy, distributed frequency control through inverter-interfaced sources (Carrasco et al., 2006) or loads (Short, Infield, & Freris, 2007) has a high potential due to the fast ramping capabilities of these devices. In any case, the task of frequency regulation will have to be shouldered by more and more small-scale and distributed devices.

From a control perspective, the main objective of frequency control is to regulate the system frequency subject to operational constraints and economic interests such as load sharing, optimal generation dispatch, or according to the outcome of reserve markets. Further constraints include a partial information structure accounting for distributed generation, liberalized markets, and limited system knowledge. A plethora of strategies has been developed to address these tasks ranging from fully decentralized to centralized architectures, partially relying on time-scale separation and hierarchical control, and being dependent on the detailed system model, load and generation forecasts. While centralized strategies such as AGC often suffer from a single point of failure, distributed or fully decentralized approaches often fall short in practical implementations and typically require a retrofitting of a costly peer-to-peer communication architecture.

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We postpone a detailed literature review to Section 2.4, where we also present some novel results of independent interest concerning robustness and fairness issues.

In this paper, we consider a nonlinear, differential–algebraic equation (DAE), and heterogeneous power system model including traditional generation, power electronic sources, and frequency-responsive as well as passive loads. We assume that the sources are already equipped with primary droop control, and we focus on designing the secondary control strategy while simultaneously solving a tertiary economic dispatch problem. Our control approach falls square in between centralized and distributed architectures, and it is motivated and developed by exploiting parallels in dual decomposition methods in optimization (Boyd, Parikh, Chu, Peleato, & Eckstein, 2010), auctions in markets (Varian & Repcheck, 2010), mean field control (Grammatico, Parise, Colombino, & Lygeros, 2016), as well as classic AGC (Machowski et al., 2008). Interestingly, our control architecture includes many previous frequency control strategies for specific parameter sets.

Specifically, we start with an online optimization routine for the steady-state dynamics based on the dual decomposition method that evaluates the price of frequency violation in feedback with the optimal generation response of each generator. Our iterative algorithm resembles a decentralized auction mechanism for a spot market. Next, we propose a continuous-time feedback control version of this optimization scheme as an aggregation of a convex combination of frequency measurements, followed by integral control and optimal local allocations of a broadcast control signal. Our gather-and-broadcast controller is such that the closed-loop equilibria of the power system are optimizers of the economic dispatch. We believe that our gather-and-broadcast control strategy combines appealing features from both centralized and distributed strategies. It robustifies the frequency control by drawing upon the information of multiple sensors and distributing the control actions to multiple generators, it does not require any model knowledge, it relies on unidirectional broadcast communication, and it is privacy preserving: no participant needs to communicate its internal model or cost function. We prove local asymptotic stability of the nonlinear closed-loop DAE system for a specific class of strictly convex cost functions that give rise to typical secondary control curves encountered in practice, including dead-bands, linear response regions, and saturation effects. The main technical results in this paper generalize those in our preliminary work (Dörfler & Grammatico, 2016), which are based on quadratic cost functions and more restrictive assumptions on the system parameters. Our analysis relies on a dissipative Hamiltonian formulation of the closed-loop system, an incremental Bregman-type Lyapunov function as in Trip, Bürger, and De Persis (2016), convex analysis (Rockafellar & Wets, 1998), and a LaSalle invariance principle for DAE systems (Dörfler & Schiffer, 2016; Hill & Mareels, 1990).

The paper is organized as follows. In Section 2, we introduce the frequency control problem that includes both frequency regulation and optimal economic dispatch, and we provide a comprehensive literature review. In Section 3, we propose our novel frequency control strategy, and in Section 4 we show local asymptotic stability of a desirable subset of the closed-loop equilibria. In Section 5, we illustrate the performance of our strategy with a simulation case study on the IEEE39 New England grid and also compare it to other controllers. Section 6 concludes the paper and raises some open questions.

*Notation*

$\mathbb{R}, \mathbb{R}_{>0}, \mathbb{R}_{\geq 0}, \mathbb{R}_{<0}, \mathbb{R}_{\leq 0}$ denote the set of real, positive real, non-negative, negative and non-positive real numbers, respectively. $A^\top \in \mathbb{R}^{m\times n}$ denotes the transpose of $A \in \mathbb{R}^{n\times m}$. Given some matrices $A_1, \dots, A_N$, diag $(A_1, \dots, A_N)$ denotes the block-diagonal matrix with $A_1, \dots, A_M$ in block-diagonal positions. Given some functions or scalars $f_1, \dots, f_N$, we use the vector notation $\boldsymbol{f} := [f_1, \dots, f_N]^\top$ and matrix notation $\boldsymbol{F} := \mathrm{diag}(f_1, \dots, f_N)$, unless differently specified. $\mathbb{1}_N$ ($\mathbb{0}_N$) denotes a vector in $\mathbb{R}^N$ with elements all equal to 1 (0). Given a function $f : \mathbb{R}^N \to \mathbb{R}$, the operator $\nabla f(\cdot) : \mathbb{R}^N \to \mathbb{R}^N$ denotes the gradient $\left[\frac{\partial f}{\partial x_1}(\boldsymbol{x}), \dots, \frac{\partial f}{\partial x_N}(\boldsymbol{x})\right]^\top$. The sum operator, i.e., $\sum_i$ or $\sum_{i,j}$, applies to all terms on its right side as in Rockafellar and Wets (1998).

## 2. Frequency control in power systems

### 2.1. Power system model

Consider a power system modeled as a graph $G = (\mathcal{V}, \mathcal{E})$ with nodes (or buses) $\mathcal{V} = \{1, \dots, N\}$ and edges (or branches) $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. With each bus $i \in \mathcal{V}$, we associate a harmonic voltage waveform $V_i \cos(\omega^* t + \theta_i)$, where $\omega^* = 2\pi \cdot f^*$ (and $f^* = 50$ Hz or $f^* = 60$ Hz is the nominal grid frequency). We consider a lossless high-voltage transmission grid with topology induced by the sparse susceptance matrix $\tilde{B} \in \mathbb{R}^{N\times N}$. We partition the buses as $\mathcal{V} = \mathcal{G} \cup \mathcal{F} \cup \mathcal{P}$ corresponding to synchronous generators $\mathcal{G}$, buses with frequency-responsive devices $\mathcal{F}$ (e.g., frequency-sensitive loads or inverter sources performing droop control), and passive buses $\mathcal{P}$ (e.g., static loads or inverters performing maximum power-point tracking). The associated DAE model reads as (Hill & Mareels, 1990; Machowski et al., 2008)

$$\forall i \in \mathcal{G} : M_i \ddot{\theta}_i + D_i \dot{\theta}_i = P_i + u_i - \sum_{j\in\mathcal{V}} B_{i,j} \sin(\theta_i - \theta_j) \quad (1a)$$

$$\forall i \in \mathcal{F} : D_i \dot{\theta}_i = P_i + u_i - \sum_{j\in\mathcal{V}} B_{i,j} \sin(\theta_i - \theta_j) \quad (1b)$$

$$\forall i \in \mathcal{P} : 0 = P_i + u_i - \sum_{j\in\mathcal{V}} B_{i,j} \sin(\theta_i - \theta_j) \quad (1c)$$

where, for all $i \in \mathcal{V}$, $P_i \in \mathbb{R}$ is a constant power injection or demand (positive for sources and negative for loads), $u_i \in \mathcal{U}_i = [\underline{u}_i, \overline{u}_i] \subset \mathbb{R}$ is a controllable injection or demand, and $B_{i,j} := \tilde{B}_{i,j} V_i V_j$ is the effective susceptance for all $i, j \in \mathcal{V}$. A generator $i \in \mathcal{G}$ is characterized by its rotational inertia $M_i > 0$ and primary droop control coefficient $D_i > 0$. A frequency-responsive device $i \in \mathcal{F}$ is characterized by its frequency-sensitivity $D_i > 0$ (e.g., the droop coefficient for inverters or actively controlled loads, or the damping of a frequency-dependent load). Passive buses (inverters performing power-point tracking and static loads) have no dynamics. Finally, the absence of integral control at node $i \in \mathcal{V}$ is modeled by $\mathcal{U}_i = \{0\}$.

**Remark 1** (*Unmodeled Dynamics*). We do not model reactive power and voltage dynamics, as they do not affect the frequency control problem on the considered time scales—though all of our forthcoming analyses can be extended under a definiteness assumption on the power flow Jacobian; see De Persis, Monshizadeh, Schiffer, and Dörfler (2016) for a related analysis. □

Finally, we note that the vector field in (1) is invariant under a rigid rotation of all angles. Accordingly, all equilibria of the power system model (1) are sets that are invariant under rigid rotations, and all properties such as uniqueness, optimality, and asymptotic stability of equilibria are to be understood modulo rotational symmetry.

### 2.2. Frequency regulation

Note that if there is a synchronized solution to (1) satisfying $\dot{\theta}_i = \omega_{\mathrm{sync}} \in \mathbb{R}$ for all $i \in \mathcal{V}$, then by summing up all steady-state