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A systemic viewpoint on the approximation of a power transmission line model

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Abstract: This paper explains, from a systemic viewpoint, the importance to take into account the dynamic structure of the whole system in order to simplify the distributed parameters model of the transmission lines. Usually, the latter is approximated without considering its connection with the rest of the system, by comparing only its input-output behaviour with the one of the simplified model. Here, it is shown that this way to do can lead to biased results. More precisely, it is shown that the short-lines hypothesis leads to a reduction link with the π-model but does not indicate clearly which dynamics have to be kept in the simplified model as well as their number. This is illustrated by considering the voltage collapse phenomenon. From this analysis, a more systemic approximation way is proposed to reduce subsystems of a general complex system. In power systems field, all these investigations can help to improve the models used for simulation and control synthesis. Especially, to better connect the models to the specific phenomena which have to be reproduced in simulation.

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1. INTRODUCTION

The voltage and the current of power electric lines are both sum of two waves which travel with same finite velocity and in opposite directions along the line. Their dynamic is modelled by a Distributed Parameters (DP) model which consists of two Partial Differential Equations (PDE) giving these quantities in function of time and space. It is thus an *infinite-dimension* system. When it is connected to the rest of the power system, it leads to a full description of all the physical phenomena resulting from the different interconnections. However, for realistic applications, such a full and detailed model of the lines is difficult to be used and need to be simplified. For instance, in the case of short and medium length lines (up to 250 Km), the propagation phenomena is generally not taken into account and a simplified model, called π-model, is used. It is thus described by ordinary differential equations giving the voltages and the currents only at the ends.

In power systems, the models above are used to simulate different phenomena. The π-model is used, e.g., in (Meyer and Stubbe, 1992) to perform load-flow computations, to analyse transient stability and to design voltage controllers. To capture the wave propagation, ElectroMagnetic Transients Program (EMTP) simulators, like (Dommel, 1969), use a delay model obtained from the trajectories of the DP one (when the frequency dependence of the parameters can be neglected). Both models are based on physical considerations. However, their adequacy to reproduce a specific phenomenon, with the overall power system, was always checked *a posteriori* by experimental tests. The main reason of this is that only the input-output behaviours of the full and the proposed simplified models are compared in order to validate the simplification. All the interactions with the other components of the system are not considered.

In this context, our goal is to establish a *systemic* link between the phenomena to be reproduced in simulation and the appropriate model of the transmission line. This firstly motivated us to investigate the dynamic properties (especially the modes and the transfer function) of both π and DP models in the setting of systems theory. This provides more informations on the relationship between their dynamic structures and their behaviours than a simple comparison of their trajectories. Indeed, the π-model was a priori adopted and it was afterwards checked that, in some particular situations, it provides trajectories comparable with the ones of the DP model. It is explained here the connection between the dynamic structures of the two models. More precisely, the π-model is shown to be close to a first order modal truncation of the DP model. Also, it is explained that the systematic use of the π-model as an approximation of the DP one, can lead to less satisfactory results in some situations when the line is connected to the rest of the system. Finally, a more systemic and appropriate way is proposed to well approximate the DP model of the transmission lines. All these investigations can be extended to other components of a power system and give a basis to further study the adequacy of the models to each typical power system dynamics (like, e.g., voltage response inter-area oscillations, sub-synchronous resonance,...).

The paper is organised as follows: in Section 2, the mathematical background used in our developments is recalled. In Section 3, the dynamic structures of both DP and π

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models of the line are analytically developed and their poles and trajectories are compared. A reduction link between them is established in the case of open lines. In Section 4, is explained how the use of the π-model, based on the hypothesis of short-lines can lead to less satisfactory results. Section 5 presents the proposed technique to well approximate the DP model of the transmission lines. Conclusions and ways in which our results can be exploited to improve general models used for simulations, analysis and control of power systems are presented in Section 6.

2. MATHEMATICAL BACKGROUND

Let Z be a Hilbert space, and $\mathcal{A} : D(\mathcal{A}) \subset Z \to Z$ a linear operator with the domain $D(\mathcal{A})$. If $\mathcal{A}$ is compact, the set of complex values $\lambda \in \mathbb{C}$ for which the operator $(\mathcal{A} - \lambda I)$ is not invertible is said the *spectrum* of $\mathcal{A}$ and it is denoted by $\rho(\mathcal{A})$. Then the set of eigenvalues of $\mathcal{A}$ is a subset of $\rho(\mathcal{A})$ for which the following equation is satisfied

$$\mathcal{A}\phi_n = \beta_n \phi_n, \tag{1}$$

where $\{\beta_n, n \geq 1\}$ are the eigenvalues of $\mathcal{A}$ and $\{\phi_n, n \geq 1\}$ the corresponding eigenvectors (eigenfunctions). Notice that n is not necessarily finite. In the case where $\mathcal{A}$ is a self-adjoint operator, $\{\phi_n, n \geq 1\}$ forms an *orthonormal basis* and then each element $z \in Z$ can be uniquely written as $z = \sum_{n=0}^{\infty} \langle z, \phi_n \rangle_Z \phi_n$ where $\langle ., . \rangle_Z$ is the inner product in Z. Otherwise, if $\mathcal{A}$ is non-self-adjoint but a Riez spectral operator, then each $z \in Z$ can be written in a unique way as $z = \sum_{n=0}^{\infty} \langle z, \psi_n \rangle_Z \phi_n$ where ψ_n are the eigenvectors of the adjoint of $\mathcal{A}$ noted $\mathcal{A}^*$.

Also, the following representation

$$\begin{cases} \dfrac{dz(t)}{dt} = \mathfrak{A} z(t), \qquad z(t)|_{t=0} = z_0, \\ \mathfrak{B} z(t) = u(t). \end{cases} \tag{2}$$

is a general form of an *abstract boundary control problem* where $\mathfrak{A} : D(\mathfrak{A}) \subset Z \to Z$ is an operator, $u(t) \in U$ the input and $\mathfrak{B} : D(\mathfrak{B}) \subset Z \to U$ is called *boundary* operator with $D(\mathfrak{A}) \subset D(\mathfrak{B})$. More explanations and details can be found, e.g., in Curtain and Zwart (1995) or Tucsnak and Weiss (2009).

3. ANALYTIC DEVELOPMENTS

To start, consider the distributed parameters model of the line. If the transverse conductance is neglected, it can be written as

$$\begin{cases} \dfrac{\partial v(x,t)}{\partial x} = -L \dfrac{\partial i(x,t)}{\partial t} - R i(x,t), \\ \dfrac{\partial i(x,t)}{\partial x} = -C \dfrac{\partial v(x,t)}{\partial t}, \end{cases} \tag{3}$$

where R, L, and C are positive constant parameters given per unit length (see, e.g., (Miano and Maffucci, 2001)). To connect (3) with the other components of the system, boundary conditions are needed. They define the inputs and the outputs of the line model and then describe the behaviour at the extremities of a line of length ℓ. Here, the line is considered open at the extremity $x = \ell$ and submitted to an ideal voltage source $v_0(t)$ at extremity $x = 0$. Thus, when the equations of (3) are combined to eliminate the current $i(x,t)$ and the boundary conditions are added, one gets the system

$$\begin{cases} \dfrac{\partial^2 v(x,t)}{\partial^2 x} - LC \dfrac{\partial^2 v(x,t)}{\partial^2 t} - RC \dfrac{\partial v(x,t)}{\partial t} = 0, \\ v(x,t)|_{t=0} = 0, \ \dfrac{\partial v(x,t)}{\partial t}|_{t=0} = 0, \\ v(x,t)|_{x=0} = v_e(t), \ \dfrac{\partial v(x,t)}{\partial x}|_{x=\ell} = 0, \end{cases} \tag{4}$$

defined for $x \in \Omega = [0 \ \ \ell]$ and $t \in [0 \ \ +\infty)$. Its first equation is called *damped wave equation* or *telegrapher's* equation. From a systemic viewpoint, (4) can be formulated as an abstract boundary control problem of form (2). Indeed, by considering $z_1(t) = v(x,t)$ and $z_2(t) = \dfrac{\partial v(x,t)}{\partial t}$ one has

$$\begin{cases} \dfrac{d}{dt} \underbrace{\begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix}}_{z(t)} = \underbrace{\begin{pmatrix} 0 & 1 \\ \alpha \dfrac{\partial^2}{\partial^2 x} & \beta \end{pmatrix}}_{\mathfrak{A}} \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix}, \\ \mathfrak{B} \begin{pmatrix} z_1(t) \\ z_2(t) \end{pmatrix} = v_e(t), \quad z(0) = 0, \end{cases} \tag{5}$$

where $\alpha = \dfrac{1}{LC}$ and $\beta = -\dfrac{R}{L}$. In the sequel, the Laplacian operator $\dfrac{\partial^2}{\partial^2 x}$ will be noted $A_0 = -\dfrac{d^2}{d^2 x}$. Now, to get the trajectories of (5), some definitions are needed. First, let us define $Z = D\left(A_0^{\frac{1}{2}}\right) \oplus L^2(\Omega)$, as the Hilbert space to which belong the solutions of (5). It is equipped with the following inner product

$$\left\langle \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}, \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} \right\rangle_Z = \left\langle \sqrt{\alpha} A_0^{\frac{1}{2}} f_1, \sqrt{\alpha} A_0^{\frac{1}{2}} g_1 \right\rangle_{L^2(\Omega)} + \langle f_2, g_2 \rangle_{L^2(\Omega)} \tag{6}$$

where $L^2(\Omega)$ is the complex space of square-integrable functions and $D\left(A_0^{\frac{1}{2}}\right)$ the definition domain of the operator $A_0^{\frac{1}{2}}$, i.e.,

$$D\left(A_0^{\frac{1}{2}}\right) = \Big\{ f \in L^2(\Omega) \,|\, f \text{ absolutely continuous}, \ \frac{df}{dx} \in L^2(\Omega) \text{ and } f(0) = 0 \Big\}.$$

Next, two conditions have to be satisfied. The first one, is that the operator $\mathcal{A} = \mathfrak{A}z$ defined by $\mathcal{A} = \mathfrak{A}|_{\mathrm{Ker}(\mathfrak{B})}$ (i.e., $\mathfrak{A}$ restricted to the kernel of $\mathfrak{B}$), for $f \in D(\mathcal{A})$, is the infinitesimal generator of a C_0-semigroup $\mathcal{S}(t)$ on Z. The second one, is that there exist an operator $\mathcal{B}$ in the set $\mathcal{L}(U, Z)$ of linear applications from U to Z, so that for $u \in U$ and $\mathcal{B}u \in D(\mathfrak{A})$, the operator $\mathfrak{A}\mathcal{B}$ belong to $\mathcal{L}(U, Z)$ and $\mathfrak{A}\mathcal{B}u = u$. For (5), both conditions are satisfied with $U = \mathbb{C}$ and $\mathcal{B} = (1 \ 0)^T$. From this point, a change of variable $h(t) = z(t) - \mathcal{B}u(t)$ leads (5) to the following *homogeneous* (i.e., $\mathfrak{B}h(t) = 0$) problem

$$\frac{dh(t)}{dt} = \mathcal{A}h(t) - \mathcal{B}\frac{du(t)}{dt} + \mathfrak{A}\mathcal{B}u(t). \tag{7}$$

Equation (7) is well posed in Z and has a unique *classical* solution (see Theorem 3.3.3 of (Curtain and Zwart,

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