Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 50-1 (2017) 10413–10418

Double Loop Control Design for Boost Converters Based on Frequency Response Data ⋆

Ricardo Alzate ** **Vilma A. Oliveira** * **Rafael F. Q. Magossi** * **Shankar P. Bhattacharyya** ***

* *Department of Electrical and Computing Engineering, Universidade de São Paulo, São Carlos, SP, Brazil (e-mail:vilma@sc.usp.br)*
** *School of Electrical Engineering, Universidad Industrial de Santander, Bucaramanga, Colombia (e-mail: ralzatec@uis.edu.co)*
*** *Department of Electrical and Computer Engineering, Texas A&M University, College Station, USA (e-mail: bhatt@ece.tamu.edu)*

Abstract: This paper presents a data driven approach for PI control design based on frequency response data and calculations of stabilizing sets. The geometrical interpretation of the loci of stability margins allows to determine the parameters of a PI controller from a space of achievable specifications, constructed from frequency response data. As illustration for the use of the proposed approach in engineering applications, regulation of the output voltage of a boost power converter circuit under a double loop control strategy is presented. Numerically, it was possible to generate enough amount of data to perform accurate predictions. The key to apply the proposed approach with success was the generation of a Bode diagram for the system with fine resolution, avoiding the explicit necessity for mathematical models.

Keywords: Boost converter, data driven, frequency response, PI control, stabilizing sets.

1. INTRODUCTION

There are two main approaches to perform control synthesis tasks. The first approach is model-based control and the control parameters are calculated using a model of the plant. Depending on the complexity of the system under study, finding a model can be cumbersome. A second approach is to extract information of the system behavior directly from data. This approach is known as data-driven and are generally model-free. The latter is an approach more suited to deal with uncertainties and unexpected changes in the structure of the system.

In relation to the case of fixed order controllers, the set of stabilizing controllers of PI and PID type can be determined based on frequency response data of the plant (Keel and Bhattacharyya, 2008). Characterization of all stabilizing fixed order controllers is critical for designing controllers which satisfy performance specifications which are based on frequency response, such as gain and phase margins (Alzate and Oliveira, 2016). This constitutes a modern approach to PI and PID control design, representing an alternative to traditional methods in general of the trial and error type.

Power converter circuits are strategic devices in energy management as they regulate power flows from sources to loads. They can be modeled as small-signal transfer functions when operating close to an operating point, allowing the use of traditional techniques for analysis and synthesis (Chen et al., 2011; Wai and Shih, 2011). One of the most popular types of power converters used in applications is the boost topology, in which the output voltage is higher than the one provided as input. However, despite its popularity, this circuit presents a structural drawback related to a non-minimum phase behavior whenever the voltage is selected as the system output, representing a challenge from the control viewpoint (Hoagg and Bernstein, 2007). Special effort has been devoted to understand and compensate the non-minimum phase feature of the boost topology under voltage mode (Bag et al., 2013). Recent developments as those reported in Lopez-Santos et al. (2015) calculate parameters for low-order controllers of power converters attending specifications with respect to stability margins.

In this paper, we explore a novel approach to design PI controllers which employs a geometrical approach to relate stabilizing controllers with a space of achievable specifications computed from frequency response data. The approach is model free and provides information about the system performance based on analytical formulations.

2. STABILIZING SETS

Let us consider the LTI plant

$$P(s) = \frac{N(s)}{D(s)} \qquad (1)$$

in cascade with a PI controller

⋆ This work was supported by the Brazilian National Council for Scientific and Technological Development (CNPq) under Ciência Sem Fronteiras Program grant 400401/2013-4, 150228/2015-5 and 150339/2016-0.

10.1016/j.ifacol.2017.08.1968

$$C(s) = \frac{k_p s + k_i}{s} \tag{2}$$

such that

$$\delta(k_i, k_p, s) = sD(s) + (k_p s + k_i)\, N(s) \tag{3}$$

represents the characteristic polynomial for an unity feedback system. The stabilizing set contains all PI controllers that stabilize the closed loop for the plant $P(s)$ and is given by

$$\mathcal{S} := \{(k_i, k_p) : \delta\left(s, k_i, k_p\right) \text{ is Hurwitz}\}. \tag{4}$$

2.1 Stability From the Signature of a Rational Function

Let us now define the following rational functions based on $\delta(k_i, k_p, s)$

$$\begin{aligned} F(s) &:= \frac{\delta(k_i, k_p, s)}{D(s)} = s + (k_p s + k_i) P(s) \\ \bar{F} &:= F(s) P(-s). \end{aligned} \tag{5}$$

The location of zeros and poles of $F(s)$ (excluding roots on the $j\omega$ axis) can be related to its net change in phase as ω varies from 0 to $+\infty$, expressed by

$$\Delta_0^{\infty} \angle F(j\omega) = \frac{\pi}{2}\sigma(F) \tag{6}$$

with $\sigma(F)$ representing the signature of $F(s)$.
The signature $\sigma(F)$ can be defined in terms of the number of zeros z_F and poles p_F of $F(s)$ located in the left (-) and right (+) half sides of the complex plane, as follows Keel and Bhattacharyya (2008)

$$\sigma(F) := \left(z_F^- - z_F^+\right) - \left(p_F^- - p_F^+\right). \tag{7}$$

The closed loop system is stable if all its characteristic roots are in the left half of the complex plane, then for stability we need $z_F^+ = 0$.

If the relative degree of $P(s)$ is $r_P = (n - m)$ for $n > m$, the degree of $\delta(s)$ is $(n+1)$ and then $z_F^- = (n+1)$. Accordingly

$$\sigma(F) = (n+1) - \left(p_F^- - p_F^+\right). \tag{8}$$

By extending the signature concept to $P(s)$ we get

$$\sigma(P) = (z_P^- - z_P^+) - (p_P^- - p_P^+)$$

and equivalently

$$\sigma(P(-s)) = \left(z_P^+ - z_P^-\right) - \left(p_P^+ - p_P^-\right). \tag{9}$$

Then

$$\sigma(\bar{F}) = \sigma(F(s)) + \sigma(P(-s) = (n+1) + (z_P^+ - z_P^-). \tag{10}$$

Now, since $z_P^+ + z_P^- = m$ it is possible to write

$$z_P^+ - z_P^- = 2z_P^+ - m \tag{11}$$

so that the signature for closed loop stability of $P(s)$ under PI control is

$$\sigma(\bar{F}) = (n+1) + (2z_P^+ - m) = n - m + 2z_P^+ + 1. \tag{12}$$

2.2 Numerical Computation of the PI Stabilizing Set

In order to analyze the effect of the PI control parameters on the poles and zeros of $F(s)$, it is convenient to separate the dependence of the real and imaginary parts of $F(j\omega)$ on k_p and k_i.
This can be achieved after multiplying $F(j\omega)$ by $P(-j\omega)$, with

$$P(j\omega) = P_r(\omega) + jP_i(j\omega) \tag{13}$$

representing the frequency response of the plant, allowing to get

$$\begin{aligned} \bar{F}(j\omega) &= F(j\omega)P(-j\omega) \\ &= \bar{F}_r(\omega, k_i) + j\omega \bar{F}_i(\omega, k_p) \end{aligned} \tag{14}$$

for

$$\bar{F}_r(\omega, k_i) := \omega P_i(j\omega) + k_i |P(j\omega)|^2 \tag{15}$$

$$\bar{F}_i(\omega, k_p) := P_r(\omega) + k_p |P(j\omega)|^2. \tag{16}$$

Now, fix $k_p = k_p^*$ and set

$$\bar{F}_i(\omega, k_p^*) = 0 \tag{17}$$

which is equivalent to

$$k_p^* = -\frac{P_r(\omega)}{|P(j\omega)|^2} =: g(\omega) \tag{18}$$

The zeros of (17) are the ω values

$$0 < \omega_1 < \omega_2 < \cdots < \omega_{l-1} < \infty^- \tag{19}$$

satisfying (18).

The signature in (12) can be alternatively expressed by (Keel and Bhattacharyya, 2008)

$$\begin{aligned} \sigma(\bar{F}) = [&i_0 - 2i_1 + 2i_2 + \cdots + \\ &+ (-1)^{l-1} 2i_{l-1} + (-1)^l \Lambda i_l]\Gamma, \end{aligned} \tag{20}$$

where

$$\Lambda = \begin{cases} 1, \text{ if } r_{\bar{F}} \text{ is even} \\ 0, \text{ if } r_{\bar{F}} \text{ is odd} \end{cases} \tag{21}$$

$$\Gamma = (-1)^{l-1} \operatorname{sgn}(\bar{F}_i(\omega_l, k_p^*)) \tag{22}$$

$$i_k = \operatorname{sgn}\left(\bar{F}_r\left(\omega_k, k_i\right)\right), \tag{23}$$

with sgn(.) as the signum function and

$$\begin{aligned} r_{\bar{F}} &= r_F + r_P \\ &= n - m + 1 \end{aligned} \tag{24}$$

being the relative degree of $\bar{F}$ and for $\omega_k, k = \{0, 1, 2, \cdots, l\}$ as in (19) with $\omega_0 = 0$ and $\omega_l = \infty^-$.

Hence, whenever the following inequality holds

$$2l - 1 + \Lambda \geq n - m + 2z_P^+ + 1 \tag{25}$$

it is possible to find combination of patterns for $i_k \in \{-1, 1\}$ achieving the signature requirement (12). These patterns allow us to solve from (23), the corresponding k_i values satisfying stability. These specifies the range over which k_p must be swept. By sweeping over k_p values and repeating the procedure we can generate numerically the stabilizing set $\mathcal{S}$ in (6).