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Modeling of low frequency dynamics of a smart system and its state feedback based active control

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ABSTRACT

Major physical systems/structures suffer from unwanted vibrations. For efficient working of such systems, these vibrations have to be controlled. In this paper, mathematical modeling of an aluminum cantilever beam with bonded multiple piezoelectric patches which act as the disturbance generator, sensor as well as control actuator has been presented. This piezoelectric laminate cantilever beam is assumed to be vibrating in a single degree of freedom i.e. in the flexural mode only and the corresponding state space models have been derived analytically using the finite element technique. Dominant modes of flexural vibration are identified from the frequency response of the developed model of the system and finally a state feedback controller based on pole placement technique is designed to actively suppress the vibrations. Through numerous simulations as well as experimental validation, the effectiveness of the active controller in damping the vibrations at various excitation frequencies as well as frequency ranges along the flexural mode is established.

1. Introduction

Low frequency structural noise and vibration is a very common problem in variety of light weight slender structures such as flexible beams. Beams form the basis of many of the mechanical as well as electromechanical structures. Recently, great progress has been accomplished in development of smart systems for active vibration control, therefore, enhancing the performance of the system dynamics. A smart system can be defined as a system fused with smart materials acting as actuators and sensors, along with a data acquisition and control system unit allowing the system to account for the unsolicited parameters/effects when excited by an external stimulus as well as amplifying the desired effects [1]. Piezoelectric materials form an important component of various smart structures/systems and have been actively studied and applied in various smart systems [2–8,44]. The simplest technique of active vibration control of smart systems is direct output feedback with measurement of displacement, velocity, accelaration, strain, etc. [9–15]. Researchers world wide have measured the output parameters using various transducers like accelerometers [14,16–19], strain gauges [20,21], laser Doppler vibrometers [22–25] apart from embedded piezoelectric patches [11,10,9,26–28]. Beam tends to vibrate in flexural, lateral and torsional modes and thus, it exhibits six degrees of freedom. [29,30]. One of the crucial parameter observed by early researchers was the bending effect of a transversely vibrating beam. Euler Bernoulli beam theory neglects the shear deformation, assuming that the cross section is perpendicular to the bending line and the plane normal to the neutral axis before deformation remains normal to the neutral axis after deformation [31].

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A beam qualifies as an Euler Bernoulli Beam if the length to height ratio is greater than 10 [32] resulting in low value of beam stiffness and thus the resonances in the system appear in a low frequency region. Modeling of low frequency dynamics like vibration of a beam using Euler -Bernoulli beam theory provides a simple approach and confer reasonable engineering approximations [33]. Finite element techniques could be applied to model smart systems as reported by [34]. From the conducted numerical studies, the authors proved that active control of the vibrating smart system was possible via constant displacement and velocity feedback by appropriate adjustment of the closed loop gain. Euler-Bernoulli equations were used by [35] to model a piezoceramic element. This smart element was then used to damp the vibrations in a cantilever beam via an optimal control law derived from Riccati equation. Analytic models of a piezoelectric laminate cantilever beam classified as a smart system which is the system of interest in this work were derived using principles of finite element modeling [9,26,27,36–38,40]. In their work, the above authors also included the dynamics of multiple piezoelectric patches used as sensor, disturbance generator and control actuator. A new μ-based control of a vibrating 3D bar with piezo-stacks actuators is designed and implemented in real time by [43] thereby highlighting the ability of the H_∞/μ-theory in controlling the occuring vibrations.

In this paper, three piezoelectric patches (PZT-5H) were bonded to an aluminum cantilever beam. This smart system was next divided into several finite elements. The dynamics of the beam under consideration i.e. vibrations, characterized by a fourth order partial differential equation were modeled by taking into account the finite element method of modeling mechanical systems and further, on application of a linear transformation which decouples the governing equation of motion of the beam in terms of principle coordinates, the multiple input-single output (MISO) mathematical model of the system was derived [9,26,27,33]. The Model was developed for first, first two as well as for first three modes of flexural vibration and the observed first resonant frequency was matched with the frequency obtained from the non parametric model. Effect of multiple resonances on the system were analyzed leading to the assertion that modeling should be restricted up to the third vibratory mode, due to the negligible effect of subsequent resonances [9,26,27]. The derived models were then used to design a state feedback controller based on pole placement technique for effective control of the system dynamics. Through successive simulations, the controller was designed for the model derived for first as well as for combination of first two modes of vibration. The closed loop poles of the system were cautiously placed so as to achieve satisfactory control on the system dynamics.

2. Mathematical modeling of the smart system

Fig. 1 depicts the smart system being divided into twenty-six finite elements before the initiation of the finite element based parametric modeling. The sensors and actuators are bonded near the fixed end of the beam i.e. to the second element, in order to sense and suppress the vibration in flexural mode only [39,40]. However, the exciter patch is attached to the fourth element in order to provide input excitation to the system. Fig. 2 depicts a smart beam element with two nodes highlighting the prominent degrees of freedom. The beam is modeled using the following assumptions [4]:

- The Cross- sections of beam, sensor and actuator remain plane and normal to the deformed longitudinal axis before and also after bending.
- Negligible addition to mass and stiffness with the employment of adhesive which bonds the sensors and actuators to the system under study.

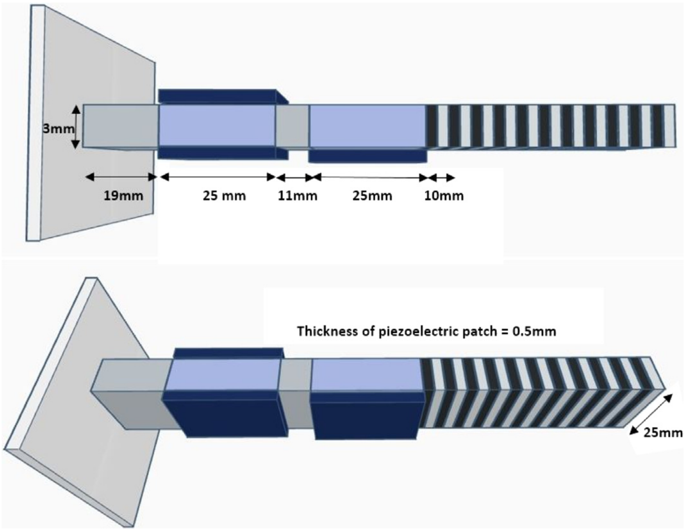

Fig. 1. 3D view of the smart beam divided into 26 finite elements.