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On the initial state of the universe in the theory of inflation

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Abstract

A toy quantum model of the inflationary universe is considered in which the role of cosmic time is played by the inflaton scalar field (its logarithm). Based on a variant of the positive energy theorem in General Relativity for the case of a closed universe, a strictly positive energy of space is introduced. The principle of minimum of the energy of space is proposed which determines a ground state as well as the excited states of the universe in quantum cosmology. According to this principle, the Beginning of the universe does exist as a state of minimal excitation of the energy of space. The initial proto-inflation quantum state of the universe is defined as a state of minimal excitation of the energy of space provided that the potential energy of the inflaton scalar field is large at the Beginning. Simultaneously, quanta of space energy excitation are introduced and the expansion of the universe can be considered as the birth of these quanta. Quantum birth of the ordinary matter becomes significant when the potential energy of the inflaton scalar field comes down to zero value.

Keywords: Universe; Theory of the inflation; Energy of space; Time; Quantum; Matter.

Introduction

The modern cosmological paradigm includes (as an inevitable part) the existence of an inflation stage with the exponential expansion of the universe [1–4]. A quantum epoch takes its own place substantially at the beginning of the universe in just the same way. Inflation theories do not explicitly specify the initial state of the universe and its size before the inflation. For instance, the inflation may be supposed to begin exactly following the quantum epoch with the Planck initial size [5]. On the other hand, in different quantum theories of the Beginning [6, 7] the classical inflation stage is considered as a natural continuation of the history of the universe. According to the Vilenkin tunneling theory [6], the de Sitter stage of the exponential expansion of the universe is sewn together with the de Sitter instanton at a definite radius determined by a vacuum energy density ρ_v in the framework of the Grand Unified Theory. In order to accommodate the tunneling theory with inflation theories where an effective scalar field (inflaton) is present, the vacuum energy density should be identified with an initial value of the inflaton potential energy. In Ref. [8], the classical stage of the inflation in the quasi-classical approximation of the Hartle–Hawking no-boundary wave function of the universe was obtained as well.

In both approaches the dynamics of the scale factor of the universe at the inflation stage was consid-

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ered as a classical one. However, quantum effects, for instance, the ordinary matter creation, are important at the final stages. This also concerns the dynamics of the scale factor of the universe. In order to formulate quantum dynamics of the scale factor, a cosmic time should be defined in the quantum universe. In Ref. [9], a canonical time parameter related to the slow-rolling inflaton scalar field was introduced in the minisuperspace model of the universe. As a result, the Wheeler–DeWitt (WDW) equation for quantum geometry [10] took the form of the Schrödinger equation with that cosmic time. This equation implies the exponential growth of the average volume of the universe, provided the initial state of the scale factor is a Gauss wave packet. The width of the packet is an arbitrary parameter in that approach.

In the present work a principle of minimal energy of space is used in order to determine the ground state of the universe and to fix the width of the initial wave packet.

In fact, in recent years a heated argument based on the chaotic theory of inflation [11] has developed on the debated topic whether there was a Beginning of the universe or whether it did not exist at all [12, 13]. Taking into account the positivity of the energy of space in a closed universe [14] we have come to the conclusion that a ground state with minimal excitation of the energy of space does exist and it can be taken as the Beginning of the universe. At the same time, the ground state is a state of maximal (Planckian) vacuum energy density. It is similar to Planck energy density in the loop quantum gravity [15], which serves as a "quantum bridge" between large classical universes, one contracting and the other expanding. The minimal energy principle was used first for definition of the ground state of the universe in Ref. [16]. Notice that this state is not stationary and evolves with time. For instance, it admits quantum fluctuations in which a universe with high initial value φ_0 of the inflaton scalar field may be nucleated. The space energy in such a proto-inflation quantum state remains minimal admitted by the Hamiltonian constraint in a closed universe.

In this paper we propose a toy model for determination of the proto-inflation quantum state of the universe and its subsequent quantum dynamics on the condition that a cosmic time related to the inflaton scalar field is introduced [9]. Together with the initial ground state of space, excited states are introduced as well, in terms of which the exponential expansion obtained in Ref. [9] is represented as quanta of space birth. It is pertinent to note that these quanta are not the same as those of a spatial volume obtained in the loop quantum gravity [17]. Here, we call quanta the excitations of the space energy.

Minisuperspace quantum inflationary model of the universe

Let us consider a homogeneous Friedman–Robertson–Walker (FRW) model of the universe with the metric

$$ds^2 = N^2(t)dt^2 - a^2(t)(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2)), \qquad (1)$$

and a scalar field ϕ with the four degree potential

$$V(\phi) = \lambda\phi^4/4$$

described by the action

$$I = \int dt\left[\frac{1}{2g}\left(\frac{\dot{a}^2 a}{N} - aN\right) - 2\pi^2 a^3\left(\frac{\dot{\phi}^2}{2N} - \frac{\lambda\phi^4}{4}\right)\right]. \qquad (2)$$

Here N is the lapse function [18], a is the scale factor of the universe, $g = 2G/3\pi$ (G is the Newton gravitational constant).

By varying the action I with respect to the lapse function N, we obtain the Hamiltonian constraint equation

$$H = \frac{1}{2}\left(\frac{gp_a^2}{a} + \frac{a}{g}\right) - \frac{1}{2}\left(\frac{p_\phi^2}{2\pi^2 a^3} + 2\pi^2 a^3\frac{\lambda\phi^4}{2}\right) \approx 0, \qquad (3)$$

where p_a and p_ϕ are canonical conjugate momenta to a and φ, respectively.

In conventional quantum theory Eq. (3) is replaced by the WDW equation for a quantum state

$$\hat{H}\psi = 0$$

with

$$p_a \to (\hbar/i)\partial/\partial a, \;\; p_\phi \to (\hbar/i)\partial/\partial\phi.$$

A problem of ordering of non-commuting multipliers (in the first round brackets) will arise in the definition of the operator $\widehat{H}$. It will be solved below.

We are interested in the slow-roll regime with the slowly varying scalar field φ. So, we shall consider the momentum

$$p_\phi = 2\pi^2 a^3\frac{\dot{\phi}}{N} \qquad (4)$$

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