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Some comments on Hurst exponent and the long memory processes on capital markets

M.A. Sánchez Granero [a,1], J.E. Trinidad Segovia [b,*,2], J. García Pérez [c,3]

[a] Department of Geometry and Topology, University of Almeria[4], Spain
[b] Department of Accounting and Finance, University of Almeria, Spain
[c] Department of Applied Economic, University of Almeria, Spain

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ABSTRACT

The analysis of long memory processes in capital markets has been one of the topics in finance, since the existence of the market memory could implicate the rejection of an efficient market hypothesis. The study of these processes in finance is realized through Hurst exponent and the most classical method applied is R/S analysis. In this paper we will discuss the efficiency of this methodology as well as some of its more important modifications to detect the long memory. We also propose the application of a classical geometrical method with short modifications and we compare both approaches.

1. Introduction

It is well known and accepted that some of the human and natural phenomena show long memory, and there are a wide amount of papers about this topic in natural sciences. In economy, as it happened with other processes observed in physics and in general in natural sciences, the study of long memory caught the interest of researchers during the seventies [20,18,19]. Ref. [21] contains many of the early papers that Mandelbrot wrote on the application of the Hurst exponent in financial time series.

Since those days, the application of the long memory processes in economy has been extended from macroeconomics to finance. Examples are Refs. [8,4,12,13,26,3,25,6] and recently Ref. [7] to quote some of them.

In finance, the discussion as to whether or not the stock market prices display long memory properties still continues since this fact has important consequences on the capital market theories. So, if stock prices show long memory this means that predictability is not a dream but a possibility. The main implication of this circumstance is that an efficient market hypothesis is clearly rejected because stock market prices do not follow a random walk.

The study of the long memory processes is normally realized through the Hurst exponent that can be estimated using three methods[5]:

* Corresponding address: Department of Accounting and Finance, University of Almeria, Carretera de Sacramento, S/N La Canada de San Urbano, 04120 Almeria, Spain. Tel.: +34 950015817; fax: +34 950633179.

E-mail addresses: misanche@ual.es (M.A. Sánchez Granero), jetrini@ual.es (J.E. Trinidad Segovia), jgarcia@ual.es (J. García Pérez).

1 Tel.: +34 950015311; fax: +34 950015481.

2 Part of this paper has been developed during the stay of Juan E. Trinidad and Miguel A. Sanchez in the Department of Finance of the University of Rotterdam, The Netherlands.

3 Tel.: +34 950015173; fax: +34 950251795.

4 Carretera de Sacramento, S/N La Cañada de San Urbano 04120 Almería.

5 Ref. [29] contains an interesting description of these methods.

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1. R/S Analysis.
2. Detrended Fluctuation Analysis.
3. Periodogram Regression.

Our main objective in this paper is to prove that the most popular method used to estimate the Hurst exponent, the R/S analysis, exhibits serious problems and as a result it is possible to obtain evidence of long memory in random series. To avoid these problems, two geometrical methods are introduced to be applied in capital markets. One of them (GM1) is based on a classical formulae of Hurst exponent while the other one (GM2) is based on a new formulae which is introduced in Section 4 and developed further in the Appendix. We will show how this methodology will allow us to study the presence of the long memory processes in the capital markets from a stricter point of view. Regarding the R/S analysis, a new approach to the application of the Anis and Lloyd's formula in [1] is introduced to solve the problems presented when subintervals of small lengths are used.

The organization of the paper is as follows: In Section 2 we introduce the Hurst exponent basis as well as the R/S analysis to calculate it. In Section 3 we study R/S analysis behavior in the particular case of random series. Section 4 presents a different approach to obtain the Hurst exponent and it is compared with R/S and the modified R/S analysis. The paper continues with an empirical application using the different approaches and finally Section 6 presents the main conclusions.

2. The Hurst exponent and long memory processes. Classical estimation via R/S analysis

The Hurst exponent is the classical test to detect long memory in time series. This analysis was introduced by English hydrologist H.E. Hurst in 1951, based on Einstein's contributions regarding Brownian motion of physical particles, to deal with the problem of reservoir control near Nile River Dam. R/S analysis in economy was introduced by Mandelbrot [18,19,21], who argued that this methodology was superior to the autocorrelation, the variance analysis and to the spectral analysis.

As we pointed out there are a wide range of papers about this topic in economy in general and in finance in particular. It is curious that most of them concluded finding evidence of memory in financial data. Examples are Ref. [10] in New York Stock Exchange, Ref. [24] in several stocks and Indexes such us S&P 500 Index, Ref. [22] in Athens Stock Exchange, Ref. [11] in S&P 500 return series before the market crashed in 1987 and 1990 or Ref. [15] in London Stock Exchange. Only Refs. [16,17] reported no evidence of long range dependence in stock returns.

The eldest and best-known method to estimate the Hurst exponent is R/S analysis. It was proposed by Mandelbrot and Wallis [20], based on the previous work of Hurst [14].

The procedure is as follows. The time series (of returns) of length L has to be divided into d sub series ($Z_{i,m}$) of length n, and for each sub series $m = 1, \ldots, d$. Then,

1. It is necessary to find the mean (E_m) and the standard deviation (S_m) of the sub series ($Z_{i,m}$).
2. The data of the sub series ($Z_{i,m}$) has to be normalized by subtracting the sample mean $X_{i,m} = Z_{i,m} - E_m$ for $i = 1, \ldots, n$.
3. Create the cumulative time series $Y_{i,m} = \sum_{j=1}^{i} X_{j,m}$ for $i = 1, \ldots, n$.
4. Find the range $R_m = \max\left\{Y_{1,m}, \ldots, Y_{n,m}\right\} - \min\left\{Y_{1,m}, \ldots, Y_{n,m}\right\}$.
5. Rescale the range (R_m/S_m).
6. Calculate the mean value $(R/S)_n$ of the rescaled range for all sub series of length n.

Considering that the R/S statistic asymptotically follows the relation $(R/S)_n \approx cn^H$, the value of H can be obtained by running a simple linear regression over a sample increasing time horizons.

$$\log\,(R/S)_n = \log c + H \log n. \tag{1}$$

When the process is a Brownian motion, H has to be 0.5, when it is persistent H will be greater than 0.5, and finally when it is anti-persistent H will be less than 0.5. For a white noise, $H = 0$, while for a simple linear trend, $H = 1$. Note that H must lie between 0 and 1.

3. Testing R/S analysis

To test the R/S analysis we have applied the Monte Carlo method making 10,000 random walk series. Table 1 contains the mean and standard deviation of the Hurst exponent considering different n values (n is the minimum length of the sub series used in R/S analysis). Results (for $n = 2$) shows that the Hurst exponent average value is 0.68 with a standard deviation of 0.02. It is clear that this alteration in average is a consequence of n value, because when n is large enough the mean value is nearer to 0.5.

As it can be observed, the length of the series influences the standard deviation obtained, but also the mean. To obtain values near to the real 0.5 it is needed to choose a large n, which is impossible for short series.

Table 2 compares the influence of the series length on the calculation of the Hurst exponent for 1000 random walks. For the R/S analysis, n were chosen so that the estimation of the mean were the most accurate (but then the standard deviation was greater).