Automatica 93 (2018) 211–223

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Distributed Pareto-optimal state estimation using sensor networks☆

Francesca Boem [a,*], Yilun Zhou [b], Carlo Fischione [c], Thomas Parisini [d,e,f]

[a] Department of Electronic and Electrical Engineering, University College, London, UK
[b] Huawei Technologies, China
[c] Automatic Control Department, KTH Royal Institute of Technology, Stockholm, Sweden
[d] Department of Electrical and Electronic Engineering, Imperial College, London, UK
[e] Department of Engineering and Architecture, University of Trieste, Italy
[f] KIOS Research and Innovation Centre of Excellence, University of Cyprus, Cyprus

ARTICLE INFO

Article history:
Received 18 January 2017
Received in revised form 15 December 2017
Accepted 27 February 2018

Keywords:
Distributed
State estimation
Prediction
Sensor
Networks
Optimal estimation

ABSTRACT

A novel model-based dynamic distributed state estimator is proposed using sensor networks. The estimator consists of a filtering step – which uses a weighted combination of information provided by the sensors – and a model-based predictor of the system's state. The filtering weights and the model-based prediction parameters jointly minimize – at each time-step – the bias and the variance of the prediction error in a Pareto optimization framework. The simultaneous *distributed* design of the filtering weights and of the model-based prediction parameters is considered, differently from what is normally done in the literature. It is assumed that the weights of the filtering step are in general unequal for the different state components, unlike existing consensus-based approaches. The state, the measurements, and the noise components are allowed to be individually correlated, but no probability distribution knowledge is assumed for the noise variables. Each sensor can measure only a subset of the state variables. The convergence properties of the mean and of the variance of the prediction error are demonstrated, and they hold both for the global and the local estimation errors at any network node. Simulation results illustrate the performance of the proposed method, obtaining better results than state of the art distributed estimation approaches.

1. Introduction

One of the fundamental applications of sensor networks is to estimate and track the state of targets or processes that are evolving in the sensing field. Useful in many monitoring scenarios, such as for example, target tracking and environment and agriculture monitoring, in sensor networks the estimations have to be distributed at each sensor node. In this paper,[1] we address the problem of distributed state estimation and prediction over sensor networks in a multi-objective optimization framework.

Given their importance, distributed estimators have been the subject of many investigations in the area of networked control (see, as example, Christofides, Scattolini, de la Pena, & Liu, 2013; Farina, Ferrari-Trecate, & Scattolini, 2010; Garin & Schenato, 2010) and distributed fault diagnosis (Boem, Ferrari, Parisini, & Polycarpou, 2011; Franco, Olfati-Saber, Parisini, & Polycarpou, 2006), among others. Generally, in these papers it is assumed that distributed estimation works according to the following procedure (Ding, Wang, & Shen, 2014; Garin & Schenato, 2010): each node in the network locally estimates the state of a common dynamic system; then, it communicates measurements and estimates only to neighboring nodes, and filters the measurements by taking a linear combination of its own and neighboring's measurements and predictions; finally, each node uses the current

☆ This work has been partially supported by the European Union's Horizon 2020 research and innovation programme under grant agreement No 739551 (KIOS CoE). This work has also been conducted as part of the research project *Stability and Control of Power Networks with Energy Storage* (STABLE-NET) funded by the RCUK Energy Programme (contract no: EP/L014343/1). The material in this paper was partially presented at the 51st IEEE Conference on Decision and Control, December 10-13, 2012, Maui, Hawaii, the 12th European Control Conference, July 17-19 2013, Zurich, Switzerland, and the 14th annual European Control Conference, July 15-17, 2015, Linz, Austria. This paper was recommended for publication in revised form by Associate Editor Erik Weyer under the direction of Editor Torsten Söderström.

* Corresponding author.
E-mail addresses: francesca.l.boem@gmail.com (F. Boem), zhouyilun1989@gmail.com (Y. Zhou), carlofi@kth.se (C. Fischione), t.parisini@gmail.com (T. Parisini).

[1] See Boem, Xu, Fischione, and Parisini (2012, 2013, 2015) for some preliminary results: in Boem et al. (2012) signal estimation is considered, while in Boem et al. (2015) the entire state is assumed to be measurable by each node.

https://doi.org/10.1016/j.automatica.2018.03.071
0005-1098/

filtered measurements to implement a model-based predictor, smoothing the previous prediction error. However, there are several aspects in this general procedure that have not yet been fully considered.

(1) The first important aspect pertains the number of accessible states. Due to geographic nodes distributions, technological constraints, etc., it can happen that, although the overall network observes the entire state, each node measures only a subset of the variables forming the overall state. We refer to this case as *partially-measurable state*. Most of the existing results have been obtained under the assumption of complete measurement information, thereby bringing much conservatism in applications (see Ding et al., 2014 for a survey about distributed filtering over sensor networks). However, if the state components are correlated, then a node could still in principle perform an accurate estimation of the state components it has not directly access to. How to perform estimation and prediction of the overall state at each node, despite partial measurement and incomplete information, has been investigated, for example, in Khan, Kar, Jadbabaie, and Moura (2010), Stanković, Stanković, and Stipanović (2009), Wu, Jia, Johansson, and Shi (2013) and Zhang, Feng, and Yu (2012).

(2) A second important aspect concerns the possible presence of bias in the measurements, an important aspect often neglected in existing approaches. Due to measurement errors, model uncertainties, and message losses, the estimates are in practice affected by bias. The bias leads to unknown statistical distribution of the estimation error. If the bias of the estimators is not considered, it may grow unbounded. Nevertheless, the performance criterion of the estimators in the literature is essentially based only on the variance of the estimation error (see Caballero-Águila, García-Garrido, & Linares-Pérez, 2014; Meng & Chen, 2014; Speranzon, Fischione, Johansson, & Sangiovanni-Vincentelli, 2008; Yang, Chen, Wang, & Shi, 2014 as examples), which leads to poor performance of the distributed estimation process when biases are present. Therefore, when designing distributed estimators, we face at least two indicators of the quality of the estimators: the mean of the estimation error (bias) and the variance. To the best of our knowledge, in this paper we present the first approach in which these two indicators are *simultaneously* taken into account. Specifically, we simultaneously minimize both the mean (the bias) and the variance of the global prediction error by posing a multi-objective Pareto optimization problem that can be solved in a distributed way by each sensor without a centralized coordination.

(3) A third important aspect is related to the fact that in the literature the filtering phase and the prediction phase are designed independently, e.g., Alriksson and Rantzer (2006), Carli, Chiuso, Schenato, and Zampieri (2008) and Stanković et al. (2009) for the sake of tractability and ease of implementation. This separation may lead to suboptimal solutions. Instead, in the paper the filtering weights and model-based prediction parameters are allowed to be time-varying and are jointly optimized by each sensor at each step, thus paving the way to improved prediction schemes compared to the state of the art.

(4) A fourth important aspect is the instantaneous performance compared to the asymptotic one. Although distributed estimators may asymptotically perform well, in the transient bias and variance of the estimation error may take on unacceptably large values. In the proposed approach – even if we show that the asymptotic convergence is achieved like in well-known Kalman-based approaches (see Olfati-Saber, 2009) – the bias and the variance of the prediction error are jointly optimized at each time-step thus showing good instantaneous performance. Convex sufficient conditions to guarantee asymptotic convergence of the estimation error mean are derived. Furthermore, the proposed approach only assumes the knowledge of the mean and variance of the process and measurement noises without need of any further assumption on their probability distributions.

To sum up, the proposed distributed estimation technique is characterized by the following main features:

(1) only a subset of the state variables are measured at each node;
(2) the mean and the variance of the estimation error are jointly minimized via Pareto optimization;
(3) the filtering and the prediction steps are jointly designed;
(4) optimized performance at each estimation step and asymptotic convergence of the estimation error mean;
(5) knowledge of mean and variance of process and measurement noises only is required.

In the following, we further elaborate on the original contributions brought about by these characteristics with respect to the literature.

1.1. State of the art

Distributed Kalman Filtering is an active area of research, see, e.g., Ding et al. (2014) and Mahmoud and Khalid (2013), where a survey about distributed filtering methods over sensor networks and distributed Kalman filtering methods are presented, respectively. Unlike Kalman filtering approaches (such as Olfati-Saber, 2009), in our study no Gaussian assumptions on the probability distribution of the measurement and modeling noises are made. Instead, we assume knowledge of the mean and covariance matrix of the noise components, without these being necessarily Gaussian. When the estimation problem we are considering is solved by a centralized approach, the Kalman filter is optimal under Gaussian assumption on the noises, and represents the best linear filter also when disturbances are non-Gaussian (Davis, 1977). However, the scenario we are facing is more challenging because we consider a distributed case, where the prediction is computed locally without the coordination of a central agent, differently from Deshmukh, Natarajan, and Pahwa (2014).

Besides distributed Kalman filtering, roughly two different approaches have been proposed to the problem of distributed state estimation and prediction. First, the approaches based on diffusion strategies, such as the ones proposed in Cattivelli and Sayed (2010) and Speranzon et al. (2008), where the diffusion of the local estimations across neighbors is applied after incremental update. These are in contrast with the second approach: consensus strategies, used, e.g., in Spanos, Olfati-Saber, and Murray (2005), where consensus is applied to obtain average observations or estimations at each filtering step. Finally, Kalman-Consensus filtering approaches have been designed (see Olfati-Saber, 2009 as example) with the objectives of estimating the state of the system and reaching a consensus with neighboring estimator agents on the estimate.

In this paper, we consider a multi-objective optimization case and we simultaneously take into account both the mean and the variance of the prediction error. This is in contrast to Caballero-Águila et al. (2014), Carli et al. (2008), Meng and Chen (2014), Speranzon et al. (2008) and Yang et al. (2014), where only minimum variance solution is studied, and from Stanković et al. (2009), where the consensus parameters minimize the steady-state mean-square prediction error. In Mitra and Sundaram (2016), Park and Martins (2017) and Wang and Morse (2017) distributed observers are designed for the case where the state is only partially observable by each sensor, but the estimation weights are designed to guarantee convergence and state omniscience properties, not optimizing bias and error variance features. In Khan et al. (2010), the considered problem for distributed estimation is similar, dealing with the design of the consensus parameters and local innovation gains to optimize a different performance criterion. Differently from the proposed method, in Khan et al. (2010) a special case is considered allowing to reformulate the problem so to obtain a