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Decentralized estimation of regression coefficients in sensor networks ☆

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ABSTRACT

Consider a wireless sensor network with a fusion center deployed to estimate a common non-random parameter vector. Each sensor obtains a noisy observation vector of the non-random parameter vector according to a linear regression model. The observation noise is correlated across the sensors. Due to power, bandwidth and complexity limitations, each sensor linearly compresses its data. The compressed data from the sensors are transmitted to the fusion center, which linearly estimates the non-random parameter vector. The goal is to design the compression matrices at the sensors and the linear unbiased estimator at the fusion center such that the total variance of the estimation error is minimized. In this paper, we provide necessary and sufficient conditions for achieving the performance of the centralized best linear unbiased estimator. We also provide the optimal compression matrices and the optimal linear unbiased estimator when these conditions are satisfied. When these conditions are not satisfied, we propose a sub-optimal algorithm to determine the compression matrices and the linear unbiased estimator. Simulation results are provided to illustrate the effectiveness of the proposed algorithm.

1. Introduction

Wireless sensor networks have attracted a great deal of attention due to their wide range of environmental monitoring, industrial monitoring, military, agriculture and health applications [1]. In a network with a fusion center (FC), the sensors sense a phenomenon, process the sensed data and pass the processed data to the FC. This is illustrated in Fig. 1. The sensors have limited communication bandwidth and are equipped with batteries with limited energy. Therefore, decentralized signal processing strategies that reduce the amount of data transmitted from the sensors are necessary to conserve bandwidth resources and prolong network lifetime.

In this paper, we consider a decentralized estimation of a non-random parameter vector observed by multiple sensors according to a linear regression model. The observation noise is assumed to be correlated across the sensors. Correlated sensor noise is usually the case when the phenomena to be measured is subject to similar interference and correlated ambient noise ([2]). Due to power and bandwidth limitations, each sensor compresses its data before transmitting it to the FC. We assume error-free transmission on the communication link between each sensor and the FC. Communication is only between the sensors and the FC, and this can be achieved using either orthogonal transmissions or a random access protocol. Based on the received messages from the sensors, the FC estimates the non-random parameter vector. Examples of applications of this model include imaging using a camera array and localization with an antenna array.

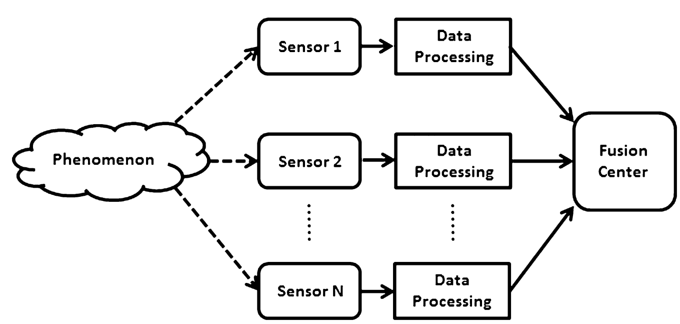

Fig. 1. Model of a wireless sensor network with a fusion center.

We model the finite bandwidth constraint as the number of real-valued messages and not as the number of binary bits (e.g., as in [3]) which can be sent from each sensor to the FC. Motivated

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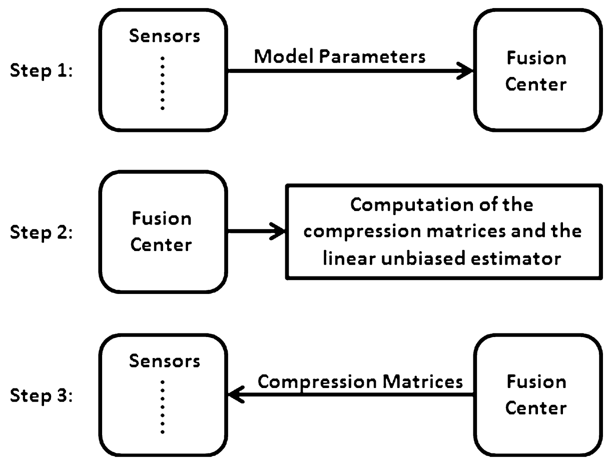

Fig. 2. The initial phase of the network.

by low-complexity requirements, we only consider linear compression functions at the sensors and a linear estimation function at the FC. The use of linear functions can be considered a special case of transform coding, similar to the discrete cosine transform or Karhunen–Loève transform in practical lossy data compression (see, e.g., [4]).

The initial phase of the network is shown in Fig. 2. The computation of the compression matrices and the linear unbiased estimator is done offline at the FC, which has full knowledge of the model parameters as in the previous examples of applications. The FC then distributes the compression matrices to the sensors.

Our goal in this paper is to design the compression matrices at the sensors and the linear unbiased estimator at the FC such that the total variance of the estimation error is minimized. We consider two scenarios. In the first scenario, the number of messages from each sensor to the FC is fixed, whereas in the second scenario, only the total number of messages from all the sensors to the FC is fixed. We identify the minimum required number of messages from each sensor to achieve the performance of the centralized best linear unbiased estimator (BLUE). Furthermore, for Gaussian signals and noise this is actually the minimum-variance unbiased estimator (MVUE). When the centralized performance is achievable, we provide the optimal compression matrices and the optimal linear unbiased estimator. When the centralized performance is not achievable, we suggest a greedy algorithm to determine the compression matrices and the linear unbiased estimator such that good estimation performance is maintained. We provide simulation results to show the effectiveness of the proposed algorithm.

This paper deals with an estimation of a non-random parameter vector. The case of random parameter vector estimation was addressed in [2,5–10]. Schizas et al. [5] used a canonical correlation analysis approach to study the problem. Grant et al. [6] reduced the problem to a rank constrained matrix approximation problem. The presence of noisy links was considered in [2]. The work presented in [10] studied the problem of jointly determining the number of messages sent by each sensor and designing the corresponding compression matrices. The works in [7–9] explored the case where the multiple access channel is coherent. In [11–14] the problem arising when the goal of the FC is to reconstruct all the sensor observations was investigated.

The case of non-random parameter vector estimation was studied in [15] and [16], assuming uncorrelated observation noise across the sensors. Here we assume that the observation noise is correlated across the sensors. Note that standard treatment of colored noise, using noise whitening, cannot be implemented in decentralized scenarios. The work presented in [16] proposed a censoring approach instead of the compression matrix based approach.

We do not assume any sparseness or compressibility on the parameter estimation vector. For more in this subject, see [17–20]. Many works allow an information exchange among the sensors (see, e.g., [21]). In this paper communication is only between the sensors and the FC.

The rest of this paper is organized as follows. Section 2 describes the model and formulates the problem. Section 3 derives the optimal solution under certain conditions that we provide and derives a sub-optimal solution when these conditions are not satisfied. Section 4 provides simulation results. Section 5 concludes the paper.

The following notations are adopted throughout this paper: a lowercase letter denotes a scalar, a lowercase boldface letter denotes a vector and an uppercase boldface letter denotes a matrix. The superscripts $(\cdot)^T$ and $(\cdot)^{-1}$ denote the transpose and inverse, respectively. The element in row i and column j of a matrix $\boldsymbol{A}$ is denoted as $\boldsymbol{A}_{ij}$. The trace and the rank of a matrix $\boldsymbol{A}$ are denoted as $\mathrm{tr}(\boldsymbol{A})$ and $\mathrm{rank}(\boldsymbol{A})$, respectively. Positive semi-definiteness of a symmetric matrix $\boldsymbol{A}$ is denoted as $\boldsymbol{A} \succeq \boldsymbol{0}$. $\mathrm{diag}(\boldsymbol{C}_1, \ldots, \boldsymbol{C}_n)$ denotes a block diagonal matrix where the matrices $\boldsymbol{C}_1, \ldots, \boldsymbol{C}_n$ are on the main diagonal. An estimator for parameter vector $\boldsymbol{\theta}$ is represented as $\hat{\boldsymbol{\theta}}$. Finally, $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ denote the set of all n-dimensional column vectors of real numbers and the set of all $m \times n$ matrices of real numbers, respectively.

2. Problem formulation

Consider a wireless sensor network consisting of a FC and N sensors, deployed to estimate a common non-random parameter vector $\boldsymbol{\theta} \in \mathbb{R}^p$. Sensor i obtains a noisy observation vector of the vector $\boldsymbol{\theta}$ according to

$$\boldsymbol{x}_i = \boldsymbol{H}_i \boldsymbol{\theta} + \boldsymbol{v}_i, \qquad 1 \le i \le N \tag{1}$$

where $\boldsymbol{H}_i \in \mathbb{R}^{n_i \times p}$ is the observation matrix, $\boldsymbol{v}_i \in \mathbb{R}^{n_i}$ is the additive noise and $\boldsymbol{x}_i \in \mathbb{R}^{n_i}$ denotes the sensor observation vector. Denote $n \triangleq \sum_{i=1}^{N} n_i$, $q_i \triangleq \sum_{j=1}^{i} n_j$, $\boldsymbol{x} \triangleq \left[\boldsymbol{x}_1^T, \ldots, \boldsymbol{x}_N^T\right]^T \in \mathbb{R}^n$, $\boldsymbol{H} \triangleq \left[\boldsymbol{H}_1^T, \ldots, \boldsymbol{H}_N^T\right]^T \in \mathbb{R}^{n \times p}$, $\boldsymbol{v} \triangleq \left[\boldsymbol{v}_1^T, \ldots, \boldsymbol{v}_N^T\right]^T \in \mathbb{R}^n$ and rewrite (1) as

$$\boldsymbol{x} = \boldsymbol{H}\boldsymbol{\theta} + \boldsymbol{v}. \tag{2}$$

The noise vector $\boldsymbol{v}$ has zero mean and covariance matrix $\boldsymbol{\Lambda} \in \mathbb{R}^{n \times n}$. Note that the observation noise is correlated across the sensors; i.e., $\boldsymbol{\Lambda}$ is not block diagonal. We assume $n \ge p$ and that the matrix $\boldsymbol{H}$ has full column rank. When the desired received signal is for example subject to external interference received by all the sensors, the noise will be correlated across sensors. Examples of this case appear in reception of astronomical signals in the presence of terrestrial interference, or when signals with a known waveform are subject to interference from other systems (e.g., when OFDM signals are received with wide-band interference).

Due to power, bandwidth and complexity limitations, sensor i compresses its observation vector to a vector $\boldsymbol{y}_i \in \mathbb{R}^{k_i} (k_i \le n_i)$ by using a linear compression matrix $\boldsymbol{C}_i \in \mathbb{R}^{k_i \times n_i}$, i.e.,

$$\boldsymbol{y}_i = \boldsymbol{C}_i \boldsymbol{x}_i, \qquad 1 \le i \le N. \tag{3}$$

The compressed data from the sensors are transmitted to the FC, assuming error-free transmission on the communication link between each sensor and the FC. Denote $k \triangleq \sum_{i=1}^{N} k_i$, $r_i \triangleq \sum_{j=1}^{i} k_j$, $\boldsymbol{y} \triangleq \left[\boldsymbol{y}_1^T, \ldots, \boldsymbol{y}_N^T\right]^T \in \mathbb{R}^k (k \le n)$, $\boldsymbol{C} \triangleq \mathrm{diag}(\boldsymbol{C}_1, \ldots, \boldsymbol{C}_N) \in \mathbb{R}^{k \times n}$ and rewrite (3) as

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} = \boldsymbol{C}\boldsymbol{H}\boldsymbol{\theta} + \boldsymbol{C}\boldsymbol{v}. \tag{4}$$

Denote $\tilde{\boldsymbol{H}} \triangleq \boldsymbol{C}\boldsymbol{H} \in \mathbb{R}^{k \times p}$, $\tilde{\boldsymbol{v}} \triangleq \boldsymbol{C}\boldsymbol{v} \in \mathbb{R}^k$ and rewrite (4) as

$$\boldsymbol{y} = \tilde{\boldsymbol{H}}\boldsymbol{\theta} + \tilde{\boldsymbol{v}}. \tag{5}$$