# Regional-scale probabilistic shoreline evolution modelling for flood-risk assessment

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## A R T I C L E   I N F O

## A B S T R A C T

Rapid deterministic modelling of shoreline evolution at regional and coastal-scheme scale enables Monte-Carlo simulations by which long-term shoreline statistics can be estimated. This paper describes UnaLinea, a fast, accurate finite difference solver of the one-line sediment continuity equation. The model is verified for the evolution of an initially straight shoreline of a plane beach subject to regular breaking waves at constant angle of incidence in the presence of either a groyne or a continuous single-point feed of sediment. Grid convergence and stability tests are used to obtain accurate, stable results, with satisfactory computational efficiency. Influences of wave input filtering and event-based sediment loading are considered. The rapid deterministic model is applied to Monte-Carlo simulations of the evolution of the west coast of Calabria, Italy for different scenarios including increased sediment load from a river and selected beach nourishment. The potential role of probabilistic shoreline evolution in regional coastal flood-risk assessment is explored through application to an idealised stretch of the Holderness coastline, U.K., where flood depths and expected damage are estimated for a 1000 year return period event.

## 1. Introduction

Beach plan-shape models are widely used in coastal engineering practice. Of these, perhaps the most popular is the one-line model derived from the mass balance of sediment in an elemental volume oriented so that its $x$-direction width lies in a direction approximately parallel to the shoreline, and its $y$-direction sides extend offshore up to the closure depth $D_c$. The model is called one-line because the beach morphology is represented by a single shoreline contour. Changes in position of this contour, together with other parameters such as wave conditions, currents, and sediment transport rates, are functions of longshore position ($x$) and time ($t$), and so the model is essentially one-dimensional in space. In the one-line model, it is assumed that the beach profile extending offshore remains constant with time. Bakker et al. [1] provide a very useful starting point for those interested in the theory and application of beach plan-shape models. They discuss the simplest one-line approach and consider profile variation along the shoreline. Horikawa [14], Komar [18], Dean and Dalrymple [6] and Reeve et al. [34] provide derivations of the one-line equation and give
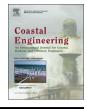
examples of its application in practice.

For small angles of wave attack, the one-line equation can be approximated by the (small angle) one-line diffusion beach response equation, originally derived by Pelnard-Considère [32] for small amplitude departures from a rectilinear coastline [7]. Analytical solutions of the Pelnard-Considère one-line diffusion equation are listed by Le Méhauté and Soldate [22], Walton and Chiu [42], Larson et al.[21], Dean and Dalrymple [6], Falqués [7], Murray and Ashton [27], Reeve et al. [34], and Reeve [33]. In practice, most one-line modelling for coastal erosion management is performed using numerical models (see e.g. [13]; [29]), due to their flexibility in modelling realistic, non-idealised coastlines that include seawalls and complicated groyne systems.

Although one-line models have been used to inform coastal management decisions for about 40 years, the use of multi-realisation simulations for probabilistic purposes is a recent development, and to date relatively little attention has been paid to their application to the generation of long-term beach response statistics [43]. To achieve multiple realisations for probabilistic analysis, a numerical one-line

model must be accurate, reliable, and computationally efficient. Such a model requires the solver to be simple yet appropriate, and that the grid spacing and time step be as large as possible while retaining accuracy and stability. Finite-difference schemes are commonly used to solve the one-line equation. Second- and higher-order explicit numerical schemes are conditionally stable in that they have a limited region of stability in terms of the time step and grid spacing (see e.g. [8]. Implicit schemes are (theoretically) unconditionally stable thus allowing much larger time-steps to be used, but are more complicated to code and can be more computationally intensive. In practice, the time-step of an implicit scheme is restricted due to the presence of numerical round-off errors [45]. One-line models are particularly prone to instability when applied to cases involving complicated infrastructure layouts and/or requiring large grids at regional scale. Although instability can be managed through intervention by a modeller undertaking a single deterministic application, this is not feasible in a probabilistic application where many thousands of individual deterministic runs may have to be realised.

This paper extends one-line modelling to probabilistic application through the use of a computationally-efficient one-line numerical model, named UnaLinea. UnaLinea has been developed with the specific intent to produce multiple, regional-scale, and long-term simulations for the probabilistic assessment of shorelines where event-based processes, such as cliff falls, artificial nourishment or fluvial loading, and their influence on coastal evolution, can be rapidly predicted. It is also demonstrated how the probabilistic application can be used to enhance the assessment of coastal flood-risk over a timescale of decades at regional scale. The structure of the paper is as follows. Section 2 describes the one-line equation and the UnaLinea numerical solver. Section 3 presents model convergence and verification test results against analytical solutions. Section 4 discusses filtering of the wave input to permit longer time-steps to be utilised, introducing the concept of "morphologically-averaged conditions". Section 5 considers regional-scale application, while Section 6 presents a demonstration case of Monte-Carlo simulation using the UnaLinea model. Section 7 describes a further application of the UnaLinea model with the RASP Structured Uncertainty (RASP-SU) flood risk model [11] within a GIS framework. The main conclusions are summarised in Section 8.

## 2. UnaLinea one-line model development

The one-line equation expresses volumetric conservation of sediment moving along the shoreline as follows (see e.g. [6]),

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + q_c = 0 \tag{1}$$

where $Q$ is the volumetric rate of alongshore sediment transport, $x$ is the distance along the shore, $A$ is the beach cross-sectional area, $t$ is time, and $q_c$ is the volume flux of material in the cross-shore direction expressed as a line source. Denoting the co-ordinate perpendicular to the beach by $y$, the beach cross-sectional area, $A$, can then be expressed as the product of $y$ and a depth $D$. If $D$ (the depth of the active profile, defined by the summation of the closure depth and the berm height) is assumed not to vary with time, then Eq. (1) can be written

$$\frac{\partial Q}{\partial x} + D \frac{\partial y}{\partial t} + q_c = 0 \tag{2}$$

Starting from some initial position, $y = y(x)$, the model evaluates successive beach positions at time intervals $\Delta t$, at points along the shore separated by $\Delta x$. So for each ordinate $x_i$ (separated from its neighbour $x_{i+1}$ by $\Delta x$) the beach position is given by $y_i(n\Delta t)$ for $n = 0$, 1, 2 ... at $t = n \Delta t$. The beach position occupies a single contour, which normally represents the high water line.

An important factor regarding model accuracy is the representation of the alongshore rate of sediment transport, $Q$, which is dominated by the action of breaking waves. For waves of small unevenness in height

along a beach with nearly straight contours, $Q$ is approximated by the CERC [4] formula. Incorporating the Osaza and Brampton (1980) term for alongshore variation in wave height, $Q$ is given by:

$$Q = H_{sb}^2 C_g \left( AK_1 \sin 2\alpha_b - \frac{B}{\tan \beta} K_2 \cos \alpha_b \frac{dH_{sb}}{dx} \right) \tag{3}$$

where $A$ and $B$ are non-dimensional parameters, $\alpha_b$ is breaking wave angle, $H_{sb}$ is significant wave height at breaking, $C_g$ is group celerity, $K_1$ and $K_2$ are sediment transport coefficients, and $\beta$ is beach slope.

Expanding Eq. (2) by substitution of $Q$ from Eq. (3) reveals that the one-line equation is an advection-diffusion equation, which is dominated by diffusion provided the wave angle is less than 45 degrees. In Unalinea, Eq. (2) is solved numerically using central differences in space and a first-order accurate forward Euler explicit scheme in time. For the small-angle approximate one-line model, where $\frac{\partial y}{\partial t} = \frac{2Q_0}{D} \frac{\partial^2 y}{\partial x^2}$ (see e.g. [21] where $Q_0$ is the amplitude of the longshore sand transport rate, then a forward-time centred-space scheme would be (theoretically) stable provided $\Delta t \leq \frac{D\Delta x^2}{4Q_0}$. This agrees with the approach taken by [20]. This criterion indicates that a doubling of the grid spacing should allow a four-fold increase in $\Delta t$. However, noting that the full one-line equation comprises advection, diffusion and possibly source-terms, it is difficult to evaluate a catch-all stability criterion *a priori*, and so we resort herein to numerical experiments (following advice from e.g. [35]).

## 3. One-line probabilistic modelling at regional scale

### 3.1. Case AS1 – single groyne

Case AS1 concerns the impact of an infinitely long impermeable groyne inserted at the middle of an initially plane beach subject to regular waves of height 1.5 m, period 6 s, at a fixed angle of incidence of 10°. The offshore depth at which waves are input is 12 m, the mean grain size of the sediment $D_{50}$ is 0.00099 m, the coefficient $K_1$ is 0.1687, the depth of active beach is 6 m, and the simulation epoch is 2 months. For a detailed coastal design study, the grid spacing $\Delta x$ would be of the order of 10 m to resolve precisely the shoreline evolution in the vicinity of the groyne. However, to achieve a rapid probabilistic 1-line model able to cover regional scales, where detailed design is not necessary, larger grid-sizes may be used to achieve high computational performance whilst ensuring that the solutions are satisfactorily accurate. A grid convergence test was undertaken using a range of grid spacing including $\Delta x = 100$, 200, 250, and 400 m. Satisfactory results were obtained for all grid-sizes, though discrepancies became noticeable for $\Delta x > 250$ m. In this case, $\Delta x = 200$ m appears appropriate for one-line modelling at regional-scale. In practice, care should be taken to carry out further grid convergence checks to ensure that a converged estimate is achieved of maximum shoreline recession and flood risk parameters. Fig. 1 shows the numerical (UnaLinea) and analytical shoreline (AS) profiles obtained after 1 month and 2 months, using the converged grid with a time step of 1 day. Note that maximum and minimum shoreline positions appear to diminish with increasing grid-spacing, a shortcoming that can be overcome, if required, by extrapolation.

We now turn to stability. For a detailed design study, hourly or 3-hourly wave conditions would typically be input in a deterministic one-line model. Much larger time-steps are desirable in a rapid probabilistic one-line model at regional scale. Fig. 2 shows the model predictions obtained for time steps varying from $\Delta t$ = day to month, compared against the analytical solution. Satisfactory predictions are achieved for $\Delta t$ = week, but divergence from the analytical solution through instability is noticeable from $\Delta t$ = fortnight and apparent throughout the model domain for $\Delta t$ = month.