Physica A 492 (2018) 2301–2309

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Research on outer synchronization between uncertain time-varying networks with different node number

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HIGHLIGHTS

- Outer synchronization between networks with different node number is researched.
- The adaptive law of the time-varying coupling matrix element is designed.
- The uncertain adjustment parameter can be determined effectively.
- The designed controller ensures outer synchronization can be achieved perfectly.

ARTICLE INFO

Article history:
Received 8 September 2017
Received in revised form 17 November 2017
Available online 28 November 2017

Keywords:
Outer synchronization
Time-varying network
Uncertain adjustment parameter
Lyapunov theorem

ABSTRACT

We research outer synchronization between uncertain time-varying networks with different node number. Based on the Lyapunov theorem, some new sufficient synchronization criteria are proposed. Further, we design the adaptive laws of the time-varying coupling matrix element and the adjustment parameter. And the designed controller can ensure that outer synchronization between uncertain time-varying networks with different node number can be achieved perfectly. At last, numerical examples are illustrated to demonstrate the efficiency of the proposed scheme.

1. Introduction

Synchronization in complex networks is a typical kind of dynamical behavior and the fundamental phenomenon. In recent years, it has been investigated extensively and many criteria for local and global synchronizations of networks have been proposed [1–6]. In literatures, many synchronization types have been reported, e.g., complete synchronization [7–9], phase synchronization [10,11], projective synchronization [12,13] and cluster synchronization [14,15], etc. Accordingly, various synchronization strategies for complex networks have been developed, e.g., the Master Stability Functions (MSF) method [16], connection graph method [17], adaptive control [18,19], pinning technique [7,20] and impulsive control [21], etc.

The above researches on synchronization of networks mainly focused on the phenomena that all nodes in a network achieve a coherent behavior, which is called inner synchronization as it is a collective behavior within a network. This kind of synchronization reveals one aspect of the real world. In reality, synchronization between/among two or more networks, which is called outer synchronization, always does exist in nature. So far, outer synchronization between networks exhibits unique application potential and becomes a new hotspot in various fields. Some typical synchronization techniques are proposed. For instance, Chandrasekar and Rakkiyappan researched impulsive synchronization of stochastic memristor-based recurrent neural networks with time delay [22]. Anbuvithya et al. investigated the non-fragile synchronization for

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https://doi.org/10.1016/j.physa.2017.11.147

bidirectional associative memory neural networks with time-varying delays and derived sufficient conditions to guarantee the non-fragile synchronization [23]. Ma et al. completed impulsive synchronization for T-S fuzzy complex dynamical networks [24]. Lü et al. achieved the outer synchronization between uncertain networks using Backstepping design [25].

In fact, there exist much uncertain factors in the real-world complex networks, such as the uncertain parameters of networks. The uncertainty will affect greatly the synchronization performance of the complex networks. For that reason, many researchers have made great effort to address this problem and some valuable results have been obtained [6,8,26]. In addition, up to now, outer synchronization of networks is focused mainly on the situation that the number of nodes in the response network is equal to that in the drive network and the topological structures of networks are certain. When two networks are used to transmit synchronously signals in practice, however, the number of nodes in the networks is often unequal. And the topological structures of the networks are often changed with time. Therefore, the research on outer synchronization between uncertain time-varying networks with different node number has a stronger practical significance. At present, the reports about this kind of network synchronization are still relatively rare.

Motivated by the above discussion, we research outer synchronization between uncertain time-varying networks with different node number. Based on the Lyapunov theorem, some new sufficient synchronization criteria are proposed. Further, we design the adaptive laws of the time-varying coupling matrix element and the adjustment parameter. And the designed controller can ensure that outer synchronization between uncertain time-varying networks with different node number can be achieved perfectly. At last, numerical examples are illustrated to demonstrate the efficiency of the proposed scheme.

The remainder of the work is organized as follows. In Section 2, some useful preliminaries are stated. Section 3 gives the main results for outer synchronization between uncertain time-varying networks with different node number. In Section 4, numerical simulations are given to illustrate the effectiveness of the proposed approaches. Finally, conclusion is presented in Section 5.

2. Network modeling and problem formulation

In this section, network modeling and problem formulation are introduced. A complex dynamical network consisting of N_1 nodes with linear couplings is considered and it is described by

$$\frac{\partial x_i(r,t)}{\partial t} = f(x_i(r,t),\theta_i) + \sigma_i \sum_{j=1}^{N_1} a_{ij} x_j(r,t) \quad (i = 1, 2, \cdots\cdots, N_1) \tag{1}$$

where r and t are space and time coordinates respectively. $x_i(r,t) \in R^n$ is the state vector of the ith node, and θ_i is the parameter. $f(x_i(r,t),\theta_i)$ is the dynamical function of ith node. $A = (a_{ij})_{N_1\times N_1}$ is the coupling matrix representing the coupling strength and the topological structure of the network, in which a_{ij} is defined as follows: if there is a connection between node i and node j, $a_{ij} \neq 0$, otherwise, $a_{ij} = 0$. The diagonal elements of matrix A are defined as $a_{ii} = -\sum_{j=1, j\neq i}^{N_1} a_{ij}$.

We take the network given by Eq. (1) as the drive network, and the response network containing N_2 dynamical nodes is given by

$$\frac{\partial y_i(r,t)}{\partial t} = f(y_i(r,t),\theta_i) + \sigma_i \sum_{j=1}^{N_2} b_{ij}(t) y_j(r,t) + u_i(r,t) \quad (i = 1, 2, \cdots\cdots, N_2) \tag{2}$$

here, $b_{ij}(t)$ is matrix element of the time-varying coupling matrix, $u_i(r,t)$ is the controller to be designed. And we assume $N_1 \geq N_2$.

We define the error between the drive network and the response network as

$$e_i(r,t) = y_i(r,t) - x_i(r,t) \quad (i = 1, 2, \cdots\cdots, N_2) \tag{3}$$

and then, the error evolution relation can be further obtained

$$\begin{aligned}
\frac{\partial e_i(r,t)}{\partial t} &= \frac{\partial y_i(r,t)}{\partial t} - \frac{\partial x_i(r,t)}{\partial t} \\
&= f(y_i(r,t),\theta_i) + \sigma_i \sum_{j=1}^{N_2} b_{ij}(t) y_j(r,t) + u_i(r,t) \\
&\quad - f(x_i(r,t),\theta_i) - \sigma_i \sum_{j=1}^{N_1} a_{ij} x_j(r,t) \\
&= f(y_i(r,t),\theta_i) - f(x_i(r,t),\theta_i) + \sigma_i \sum_{j=1}^{N_2} a_{ij} e_j(r,t) + \sigma_i \sum_{j=1}^{N_2} (b_{ij}(t) - a_{ij}) y_j(r,t) \\
&\quad - \sigma_i \sum_{j=N_2+1}^{N_1} a_{ij} x_j(r,t) + u_i(r,t).
\end{aligned} \tag{4}$$