



Option-implied objective measures of market risk

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ARTICLE INFO

Article history:

Received 7 December 2016

Accepted 24 November 2017

Available online 6 December 2017

JEL classification:

D81

D84

G01

G32

Keywords:

Risk measure

Risk dynamics

Risk-neutral densities

Value at risk

Expected shortfall

ABSTRACT

Foster and Hart (2009) introduce an *objective* measure of the riskiness of an asset that implies a bound on how much of one's wealth is 'safe' to invest in the asset while (a.s.) guaranteeing no-bankruptcy. In this study, we translate the Foster–Hart measure from static and abstract gambles to dynamic and applied finance using nonparametric estimation of risk-neutral densities from S&P 500 call and put option prices covering 2003–2013. The dynamics of the resulting 'option-implied Foster–Hart bound' are assessed in light of other well-known option-implied risk measures including value at risk, expected shortfall and risk-neutral volatility, as well as high moments of the densities and several industry measures. Rigorous variable selection reveals that the new measure is a significant predictor of (large) ahead-return downturns. Furthermore, it grasps more characteristics of the risk-neutral probability distributions in terms of moments than other measures and exhibits predictive consistency. The robustness of the risk-neutral density estimation is analyzed via Monte Carlo methods.

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The price which a man whose available fund is n pounds may prudently pay for a share in a speculation... (Whitworth 1870, p.217)

1. Introduction

Foster and Hart (2009) introduce a concept that relates the riskiness of a given gamble to the share of one's wealth up to which it is 'safe' to enter that gamble. A higher investment is 'not safe' in the sense that it results in risk exposures that exhibit a positive probability of bankruptcy in finite time. Conversely, safe investments (a.s.) guarantee no-bankruptcy. Importantly, the Foster–Hart risk measure is law-invariant; i.e. it depends only on the underlying distribution and not on the risk attitude of the investor. In this sense Foster and Hart (2009) refer to it as 'objective' and 'operational'.

Thus far, despite its interesting theoretical properties, the Foster–Hart criterion for no-bankruptcy has not been applied much in finance.¹ In this paper, we propose a novel application of the measure using an option-implied (hence forward-looking) perspective on the stock market, in order to evaluate the resulting option-

implied risk measure with respect to its predictive significance and consistency. Thus, we translate the Foster–Hart criterion from abstract gambles to applied market dynamics, using nonparametric estimation of risk-neutral densities from S&P 500 call and put option prices covering 2003–2013. In our context, the underlying decisions are purchases of stocks, which represent scalable gambles. Therefore, the appropriate interpretation of the Foster–Hart criterion is in terms of a "bound" (between zero and one) that defines the share of one's wealth that is safe to invest. (Henceforth, we shall write 'FH' as shorthand for the Foster–Hart measure of riskiness in this bounds/shares interpretation.) There are additional technical aspects to consider in this setup compared with the original formulation of Foster and Hart (2009) as the relevant gamble is both *continuous* and *dynamic*.²

We shall address the empirical question of *how much of one's wealth can one, in the sense of Foster and Hart (2009), safely invest in the S&P 500 stock index*. Obviously, the answer to *how much is safe to invest* is not straightforward, because the pig in the poke regarding such real-world investment decisions is the underlying probability distribution of the stock market, which is unknown – not only to the decision-maker but also to us as scientists. Fundamental to our analysis is therefore a formulation

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¹ Exceptions include (Bali et al., 2011; 2012; Kadan and Liu, 2014; Anand et al., 2016).

² The original operationalization by Foster and Hart (2009) was recently generalized to our setting by Riedel and Hellmann (2015) and Hellmann and Riedel (2015).

of probability distributions for the underlying gamble, which in our case concerns developments of the S&P 500 stock index over some finite horizon. One way to approach the estimation of the density function is to employ historical return distributions in combination with a dynamic model, as done in [Kadan and Liu \(2014\)](#) and [Anand et al. \(2016\)](#). Both papers confirm objective measures as important indicators of market risk. [Kadan and Liu \(2014\)](#), in particular, identify the crucial importance of higher moments, which will be an important aspect of our analysis too.

Another approach is via estimation of probability distributions based on options prices. The economic rationale for choosing the option-implied approach over historical return distributions is that options are inherently forward-looking. Only a few papers have gone down this route so far. [Bali et al. \(2011\)](#) propose a generalized measure of riskiness nesting those of [Aumann and Serrano \(2008\)](#) and [Foster and Hart \(2009\)](#). Their measure is shown to significantly predict risk-adjusted market returns, and in some cases even outperforms standard risk measures – importantly, however, the standard risk measures are evaluated only historically, not option-implied, which makes conclusive comparison of risk measures difficult. [Bali et al. \(2012\); \(2015\)](#) build on [Bali et al. \(2011\)](#), finding a positive relation between time-varying riskiness and expected market returns.

In this study, we evaluate the performance of option-implied objective risk measures as compared with other well-known option-implied risk measures including value at risk, expected shortfall and risk-neutral volatility, as well as with high moments of the densities and several industry measures. In order to do this, we need to extract full risk-neutral densities (RNDs) from the information contained in the options data (here, on the S&P 500 stock index). To get most information out of the options data (in particular regarding the high moments and tails of the distribution), our estimation is done nonparametrically using a variant of the method by [Figlewski \(2010\)](#) as introduced in [Leiss et al. \(2015\)](#). Based on day-by-day RNDs, we assess option-implied objective market risk (FH), value at risk (VaR), expected shortfall (ES) and risk-neutral volatility (RNV), and compare these with other widely used risk measures including the volatility index (VIX) and the spread (TED) between the London Interbank Offered Rate (LIBOR) and Treasury bills (T-Bill) as a measure of credit risk. Option-implied objective market risk indicators turn out to be predictively fruitful, especially in predicting large market downturns.

Relative to the existing work on option-implied objective measures of riskiness, we make three novel contributions. First, we compare option-implied FH with other option-implied objective risk measures such as value at risk and expected shortfall, rather than only with historical ones. We believe this establishes a level playing field, as predictive differences depend on the measures only and not on the information that is used to evaluate them. Moreover, our estimation of the full RNDs (instead of only moments as in [Bali et al., 2011; 2012; 2015](#)) allows an assessment of virtually any option-implied risk measure or density characteristic including – importantly – characteristics of the tails. Thus, we are able to evaluate the usefulness of a risk measure conditional on the underlying information set.

Second, while one may control for a large number of possible variables in the empirical analysis, existing studies only involve few covariates at a time ([Bali et al., 2011; 2012; 2015](#)). This is because many variables exhibit large correlations that are difficult to handle in standard statistical analysis. By contrast, our analysis offers a rigorous variable selection based on the least absolute shrinkage and selection operator (lasso, [Tibshirani, 1996](#)). The lasso performs shrinkage of regression coefficients via regularization, thus allowing systematic model selection also in the case of highly correlated covariates ([Hastie et al., 2009](#)).

Finally, we address the dynamic feature of option-implied information as the time to maturity diminishes. By contrast, [Bali et al. \(2011\); \(2012\); \(2015\)](#) use the smoothed volatility surface by OptionMetrics, which interpolates the raw options data so that the windows of forward-looking remain of constant lengths. While this smoothed surface is preferable for most scientific enquiries (hence the popularity of that data in the literature), we are particularly interested in the dynamic component of FH, to which the theoretical work by [Hellmann and Riedel \(2015\)](#) recently opened the door. We therefore use the non-smoothed dynamic ‘raw’ options data (provided by Stricknet).

Our main findings summarize as follows. First, our analyses suggest that FH provides an investor with additional information beyond standard risk measures. Second, FH is shown to be a significant predictor of large return downturns. Third, by contrast to standard risk measures, FH captures a large number of characteristics (including higher moments) of the risk-neutral probability distributions. Fourth and finally, we evaluate a form of time-consistency of the risk measures and find FH to be predictively consistent.

The remainder of this document is structured as follows. Next, we formally introduce and discuss FH in [Section 2](#), and turn to the estimation of RNDs in [Section 3](#). [Section 4](#) contains the analysis. Finally, [Section 5](#) concludes.

2. Foster–Hart riskiness

2.1. No-bankruptcy

When applying [Foster and Hart \(2009\)](#) finance, it will prove useful to work within the setup where the decision maker is allowed to take any proportion of the offered gamble. In our case the gamble g consists of buying some multiple of the risky asset at price S_0 , holding it over a period $T > 0$ and finally selling it at price S_T . Including dividends, we may define g as the absolute return $g := S_T + Y - S_0$, where Y is the monetary amount of dividends being paid over the period. This allows us to define the Foster–Hart bound $FH \in (0, 1)$ for a gamble with positive expectation as the zero of the equation

$$\mathbb{E}[\log(1 + rFH)] = 0, \quad (1)$$

with $r := g/S_0 = (S_T + Y - S_0)/S_0$ being the relative return. Since in reality any risky asset might default, FH is bounded from above by 1. [Riedel and Hellmann \(2015\)](#) show that there exist gambles for which [Eq. \(1\)](#) has no solution $FH \in (0, 1)$, even if the expected return is positive. In this case we may consistently set FH to one, $FH = 1$.

FH connects to the original definition of the Foster–Hart objective measure of riskiness as a wealth level R simply as $FH = S_0/R$ ([Foster and Hart, 2009](#), p. 791). Varying between 0 and 1, one may interpret it as the fraction of wealth at which it becomes risky to invest in the asset. Formally, this may be expressed via a no-bankruptcy criterion. Following [Foster and Hart \(2009\)](#), we define no-bankruptcy as a vanishing probability for ending up with zero wealth when confronted with a sequence of gambles

$$\mathbb{P}\left[\lim_{t \rightarrow \infty} W_t = 0\right] = 0. \quad (2)$$

[Foster and Hart \(2009\)](#) (Theorem 2) show that no-bankruptcy is guaranteed if, and only if, the fraction of wealth invested in the risky asset is always smaller than FH. In this case, wealth actually diverges; $\lim_{t \rightarrow \infty} W_t \rightarrow \infty$ (a.s.).

2.2. Growth rates

FH can be interpreted as the limit between the positive and negative geometric means of the gamble outcomes. A simple ex-

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