Physica A 492 (2018) 1317–1326

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Inequality measures for wealth distribution: Population vs individuals perspective

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HIGHLIGHTS

- Several inequality measures for wealth distribution are analyzed and their pitfalls discussed.
- A generalization of a wealth model based on pairwise transactions is proposed.
- Inequality measures for a given snapshot instant of the population overestimate individuals' wealth inequality over time.
- An alternative focus on a minimum threshold for a standard of living is explored.

ARTICLE INFO

Article history:
Received 11 May 2017
Received in revised form 30 September 2017
Available online 24 November 2017

Keywords:
Inequality measures
Wealth distribution
Econophysics

ABSTRACT

Economic inequality is, nowadays, frequently perceived as following a growing trend with impact on political and religious agendas. However, there is a wide range of inequality measures, each of which pointing to a possibly different degree of inequality. Furthermore, regardless of the measure used, it only acknowledges the momentary population inequality, failing to capture the individuals evolution over time. In this paper, several inequality measures were analyzed in order to compare the typical single time instant degree of wealth inequality (population perspective) to the one obtained from the individuals' wealth mean over several time instants (individuals perspective). The proposed generalization of a simple addictive model, for limited time average of individual's wealth, allows us to verify that the typically used inequality measures for a given snapshot instant of the population significantly overestimate the individuals' wealth inequality over time. Moreover, that is more extreme for the ratios than for the indices analyzed.

1. Introduction

Economic inequality plays, nowadays, an important role in both political and religious agendas. During the 2016 US presidential campaign, this topic was raised and debated more than once. Recently, while visiting Bolivia, Pope Francis said: "Working for a just distribution of the fruits of the earth and human labor is not mere philanthropy. It is a moral obligation". Despite economic inequality being a certainty, there is no consensus on the best way to measure it. There are inequality measures based on income, where the underlying idea is that what matters is the ability to attain a living standard by means of its own resources [1]. On the other hand, inequality measures based on consumption, claim that the present standard of living is the most important as compared to the way it is attained [2]. Finally, despite often ignored, wealth distribution

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https://doi.org/10.1016/j.physa.2017.11.059
0378-4371/

plays an important role on describing economic inequality [3]. Inequality measures based on each of these dimensions – income, consumption, and wealth – shed light on economic inequality from different perspectives, and thus all are useful for understanding the big picture. Wealth is, in general, the most unequally distributed of the three, consumption the least [4]. In this study, wealth distribution is considered for discussing inequality measures and their pitfalls.

Wealth inequality is typically assessed by considering a snapshot of the wealth distribution at a given time instant. Regardless of the measure used, it only acknowledges the momentary population inequality. Thus, the evolution over time of a given inequality measure represents the population evolution but fails to capture the individual evolution over time. For two different time instants, a given inequality measure may be the same (for the population) but individual outcome may vary significantly. Thus, the wealth ordering of the individuals may vary drastically.

It is of the utmost interest to inquire, in the context of wealth distribution for a given population, the extent to which the value of several inequality measures is significantly changed when they are applied on the mean of each individual wealth, in the last set of $n > 1$ time instants, instead of solely inspecting the (population) wealth inequality at the last nth time instant. Since, by taking the mean, the most extreme values in current wealth are expected to be attenuated, it is only reasonable to ascertain that the value for a certain inequality measure should diminish when it is based in the individuals' means. The fact that, under certain assumptions, wealth distribution seems to stabilize in equilibrium shapes can be misleading since individuals suffer constant variations from time to time, specially in the upper tail, and their ordering can be changed even though the macroscopic distribution of wealth may appear to remain unchanged. This could let us wondering if the focus should not shift from the significance of inequality between individuals in a single time instant (population) to the acknowledgment of individuals inequality over time or even to the perception of the proportion of individuals that remain below a minimum level of wealth for a long time, jeopardizing a reasonable standard of living. This issues will be explored in this paper.

Real data has been used to study inequality (see, e.g. [5,6]). However, an obvious drawback of our approach is the difficulty of using real data. Legal and confidentiality issues turn it impossible to trace the path of each individual for a given time frame. An alternative is to resort to data provided by a given theoretical model, where the simulated wealth path of each individual is explicitly known. This enables the evaluation of the extent to which the inequality is effectively smaller at individuals level as compared to the traditional population level analysis. The fact that theoretical models often state explicitly the symmetry between agents accentuates the temporary character of each individual position in the distribution and the possibly elusive nature of measures based in the wealth at a single moment. On the other hand, this fact suggests that these models may departure from the perception that in real distributions, individual positions may be more persistent.

A generalization of a simple addictive model is proposed for which we verify that, for limited time average of individual's wealth, the typically used inequality measures for a given snapshot instant of the population significantly overestimate the individual's wealth inequality over time.

The remainder of the paper is organized as follows. In the next section we present the wealth theoretical model used. In section three we describe the inequality measures considered. Computational tests are presented in section four. In the last section we have the conclusions.

2. Wealth theoretical model

Historically, the first study of income (and wealth) distribution was made by Pareto [7]. Pareto show that the power law is well fitted only in the upper tail. R. Gibrat assumed that wealth and income dynamics are based in multiplicative stochastic processes which results in the lognormal law [8]. This, however, is non-stationary. More recently, a variety of different models have been proposed considering monetary wealth (given the relative difficulty to measure non-monetary wealth). In these models, individuals' wealth evolution is based mostly in pairwise transactions resulting in money transfer from one to another. The primal assumption made is that total money in the economy is constant and as such is a conserved quantity. That is, it plays in Economics a similar role to the one played by energy in Statistical Mechanics. These type of wealth distribution models were firstly proposed by Angle [9–13]. Many physicists have proposed similar-type models including Ispolatov, Krapivsky and Redner [14], Bouchaud and Mézard [15], Chakraborti and Chakrabarti [16] and Drăgulescu and Yakovenko [17].

In this study, we follow the model in Drăgulescu and Yakovenko [17], a simple model of a closed economic system where the total money is conserved and the number of economic agents is fixed. Let us assume that there are N agents each of which with an initial amount of money equal to M. In this model, transactions between two agents consider that the quantity lost by one is equal to that earned by the other (money is locally conserved in each transaction). Thus, at a given time t, the transition equations for the exchange of wealth between a pair of agents are:

$$\begin{cases} x_i^t = x_i^{t-1} - \Delta_m \\ x_j^t = x_j^{t-1} + \Delta_m, \end{cases} \tag{1}$$

where $\Delta_m = \epsilon \times M$, with ϵ a random uniform variate in [0, 1]. Notice that if $x_i^{t-1} < \Delta_m$, i.e. if agent i does not have enough money to pay agent j, then the transaction does not occur. In order to speed up the process, at each time (iteration) t, $N/2$ transactions take place. This does not necessarily imply that transactions occur for every agent at a given time since each pair of agents is chosen randomly at a time, being randomly decided the "winner" and the "looser". The idea that there