A revisit to the dependence structure between the stock and foreign exchange markets: A dependence-switching copula approach

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A B S T R A C T

This paper develops a dependence-switching copula model to examine dependence and tail dependence for four different market statuses, namely, rising-stocks/appreciating-currency, falling-stocks/depreciating-currency, rising-stocks/depreciating-currency, and falling-stocks/appreciating-currency. The model is then applied to daily stock returns and exchange rate changes for six major industrial countries over the 1990–2010 period. The dependence and tail dependence among the above four market statuses are asymmetric for most countries in the negative correlation regime, but symmetric in the positive correlation regime. These results enrich the findings in the existing literature and suggest that analyzing cross-market linkages within a time-invariant copula framework may not be appropriate.

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1. Introduction

Measuring dependence and tail dependence is important since it helps international investors to manage risks in their portfolios. As to the direction of the dependence between the stock and foreign exchange markets, plausible arguments can be made for either a negative or positive correlation, owing to either the return chasing effect or the portfolio rebalancing and exchange rate exposure effects, respectively (Hau and Rey, 2006).[1]

The conventional Pearson correlation is not appropriate when it comes to measuring the dependence across financial markets since it gives an equal weight to both positive and negative returns as well as large and small realizations.[2] It may also lead to a significant underestimation of the risk from joint extreme events (Poon et al., 2004; Tastan, 2006). To address the above-mentioned concerns, some researchers have applied multivariate-GARCH models (Ang and Chen, 2002; Tastan, 2006; Dungey and Martin, 2007), or Hamilton's regime-switching models (Ang and Bekaert, 2002; Ang and Chen, 2002) to model the joint dynamics of returns. These articles, however, fail to examine asymmetric tail dependence between two markets since they assume that innovations follow a symmetric multivariate normal or Student-*t* distribution (Patton, 2006; Garcia and Tsafack, 2011). The extreme-value approach is another approach to anomaly detection, which focuses on the asymptotic value of the exceedance correlation (Longin and Solnik, 2001; Bae et al., 2003; Poon et al., 2004; Hartmann et al., 2004; Cumperayot et al., 2006). Extreme value theory, however, assumes an asymptotic dependence which may lead to a serious overestimation of financial risks (Poon

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[1] Return chasing effects occur when a booming domestic stock market attracts international investors, which in turn leads to an appreciation of the booming country's currency (i.e., a decrease in the nominal exchange rate which is defined as the number of units of national currency per US dollar). In such a case, the high stock price coexists with a low exchange rate, thus producing a negative correlation between the stock and foreign exchange markets. By contrast, portfolio rebalancing effects occur when the boom in domestic stock prices encourages investors to cash in their holdings and to move their capital to other countries whose markets offer better bargains. This will depreciate the booming country's currency and hence increase its exposure to exchange rate risks. High stock prices can, in this way, coexist with currency depreciation (i.e., an increase in the exchange rate), which then gives rise to a positive correlation between the two markets.

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[2] The Pearson correlation coefficient is computed as an average of the deviations from the mean. It assumes a linear relationship.

et al., 2004). This approach also requires discretion in defining extreme observations (Diebold et al., 2000; Rodriguez, 2007).

A number of previous studies have applied copulas to model co-movements across international financial markets (Cherubini et al., 2004; Patton, 2006; Hu, 2006; Ning, 2010). A copula is a multivariate cumulative distribution function whose marginal distributions are uniform on the interval [0, 1]. The copula allows one to measure the dependence structure of multivariate random variables. Several articles argue that a time-invariant copula is not appropriate and hence allow the parameters in a copula function to change over time (Van den Goorbergha et al., 2005; Patton, 2006; Harvey, 2010; Busetti and Harvey, 2011) or allow the copula function to change over time (Okimoto, 2008; Rodriguez, 2007). To model dependence switching between financial markets, the latter approach is better than the former approach since time-varying parameters in a copula function do not necessarily imply dependence switching between positive and negative regimes.

Rodriguez (2007) examined how contagion affects stock markets during the period of a financial crisis. Okimoto (2008) investigated the co-movement of stock returns across countries. These two papers allow Markov-switching on the intercept and the variance of residuals in a return function as well as the dependence and tail dependence of a copula function.[3] The current approach, however, has two unavoidable limitations. First, the assumption of the same regime classification in two different markets is imposed. In other words, if a domestic country is in a good state then the foreign country must also be in a good state. This assumption is likely to hold if we discuss the co-movement between stock markets or between foreign exchange markets. However, it is not likely to hold if our focus is to analyze the dependence between the stock and foreign exchange markets. Secondly, the technical sophistication of this approach lies in its allowing of the state variable to influence parameters in both marginal distributions and a copula function. The large number of parameters makes it difficult to find the numerical maximum of the likelihood function (Patton, 2004, 2006). To minimize the number of parameters to be estimated, Rodriguez (2007) and Okimoto (2008) simplified the specification of their marginal models.

This paper points out that the conditional correlation between the foreign exchange and stock markets switches between positive and negative correlation regimes depending on the dominance of the portfolio rebalancing effect or the return chasing effect. To characterize the above dependence-switching phenomenon, this article adopts a dependence-switching copula model to investigate the dependence structure between the two markets. The switching between the positive and negative correlation regimes depends on an unobserved state variable. A positive correlation between the stock and foreign exchange markets indicates that both stock returns and exchange rate changes move in the same direction (increasing and decreasing in tandem, so that a bull market is associated with currency depreciation, and a bear market is associated with its appreciation). Conversely, a negative correlation regime refers to a period where a bull (bear) market synchronizes with currency appreciation (depreciation). Instead of emphasizing discrete changes in the dependence and tail dependence of a single copula, this paper places emphasis on the switch in copula functions.

The advantages of our model are manifold. First, by mixing the Clayton copula with the Survival Clayton copula (where the former captures the left-side tail dependence, and the latter captures the right-side tail dependence), this paper allows for asymmetric instead of symmetric tail dependence. Second, by allowing the dependence structure across the stock and foreign exchange markets to switch between positive and negative correlation regimes during the sample period, this article closely matches the real world, in which the dependence may change. By contrast, conventional copulas capture the conditional correlation between two markets over the whole period and hence fail to allow for dependence switching. Third, our model is able to measure tail dependence among different market statuses such as rising-stocks/appreciating-currency, falling-stocks/depreciating-currency, rising-stocks/depreciating-currency, and falling-stocks/appreciating-currency. However, conventional single-copula models, extreme-value models and Markov-switching models are unable to provide the above-mentioned measures of tail dependence. In short, this paper proposes a new copula model that allows for a state-varying dependence, and then applies the model to shed light on the dependence structure between the stock and foreign exchange markets. To the best of our knowledge, this is the first paper that applies a dependence-switching copula model to investigate the dependence structure between the stock and foreign exchange markets.

The estimation of the model is complicated since the unobserved state variable affects both marginal models and the copula function. To avoid the previous problem of parameter dimensionality in estimation, we follow the suggestion of Li (2005) to proxy the unobservable state with an instrument, and then estimate marginal models with the quasi-maximum likelihood estimation method proposed by Bollerslev and Wooldridge (1992). The dependence switching copula is then fitted to the residuals from marginal models.

By applying daily stock returns and exchange rate changes for six major industrialized countries over the 1990–2010 period, this paper estimates the dependence and tail dependence for four different market statuses. These dependences are asymmetric, for most countries, in the negative correlation regime, but symmetric in the positive correlation regime. These results suggest that analyzing cross-market linkages within a time-invariant copula framework may not be appropriate. They also have important implications for cross-market risk management and international asset pricing.

The remainder of this article proceeds as follows. In Section 2 we develop the dependence-switching copula model and describe its estimation strategy. In Section 3 we discuss the empirical results. Conclusions are given in the last section.

2. The dependence-switching copula model

2.1. Copula specification

The purpose of this paper is to develop a dependence-switching copula model. A copula is a multivariate cumulative distribution function whose marginal distributions are uniform on the interval [0, 1]. It captures the dependence structure of a multivariate distribution. According to Sklar's (1959) theorem, a bivariate joint cumulative distribution function (F) of exchange rate changes ($R_{1,t}$) and stock returns ($R_{2,t}$) can be decomposed into two marginal cumulative distribution functions (F_1 and F_2) and a copula cumulative distribution function (C) that completely describes the dependence structure between the two series:

$$F(R_{1,t}, R_{2,t}; \delta_1, \delta_2, \theta^c) = C(F_1(R_{1,t}; \delta_1), F_2(R_{2,t}; \delta_2); \theta^c), \qquad (1)$$

where $F_k(R_{k,t}; \delta_k)$, $k = 1, 2$, is the marginal cumulative distribution function of $R_{k,t}$ and δ_k and θ^c are the parameter sets of $F_k(R_{k,t}; \delta_k)$ and C, respectively.

Assuming that all cumulative distribution functions are differentiable, the bivariate joint density is then given by

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[3] The two states considered in Rodriguez (2007) and Okimoto (2008) are a normal regime and an extreme regime, respectively.