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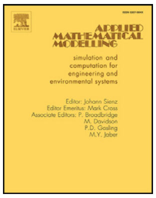

Modeling the impact of discretizing rotor angular position on computation of field-oriented current components in high speed electric drives

Leszek Jarzebowicz

Faculty of Electrical and Control Engineering, Gdansk University of Technology, Narutowicza St. 11/12, 80-233 Gdansk, Poland

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ABSTRACT

Modern drives consist of alternating current electric motors, and the field-oriented control (FOC) of such motors enables fast, precise, and robust regulation of a drive's mechanical variables such as torque, speed, and position. The control algorithm, implemented in a microprocessor, requires feedback from motor currents, and the quality of this feedback is essential to a drive's control properties. Motor phase currents are sampled and processed in order to extract their mean over a digital control interval. Afterwards, the mean phase currents are transformed into a rotating field-oriented reference frame to enable controlling the mechanical variables. The field-oriented frame rotates continuously, but in practice the transformation is carried out using a discrete angular position. This paper investigates how the discretization impacts the computed field-oriented currents in high speed drives, where the rotor displacement during a control interval is substantial. A continuous-time model of field-oriented currents is indicated as a reference to quantify errors. An original approach to normalize variables and to solve the model analytically is proposed in order to investigate how the errors related to rotor position discretization are influenced by drive operating conditions. The analytical solution is validated by computer simulation. The results show that the currently applied methodology of computing field-oriented current components, due to an invalid assumption, introduces errors of a few percent when a drive operates at high speed. These errors can be compensated using the presented solution.

1. Introduction

Digitally controlled electric drives are present in numerous devices and systems, and they have to face increasing demands regarding control precision, efficiency, and robustness [1]. There are applications such as electric vehicles or home appliances that combine the above-listed general demands with a requirement of extended speed range operation [2]. Due to the progress in control methods and motor designs, drive operational speed range has extended, for example the maximal speed of an electric drive applied to the latest Toyota Prius reaches 13,500 rpm [3], which is more than double compared with the first generation of Prius introduced in 1997.

Modern drives consist of alternating current (AC) electric machines like induction motors (IM) or permanent magnet synchronous motors (PMSM), with such motors enabling fast, precise, and robust control of mechanical variables. The

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E-mail address: leszek.jarzebowicz@pg.gda.pl

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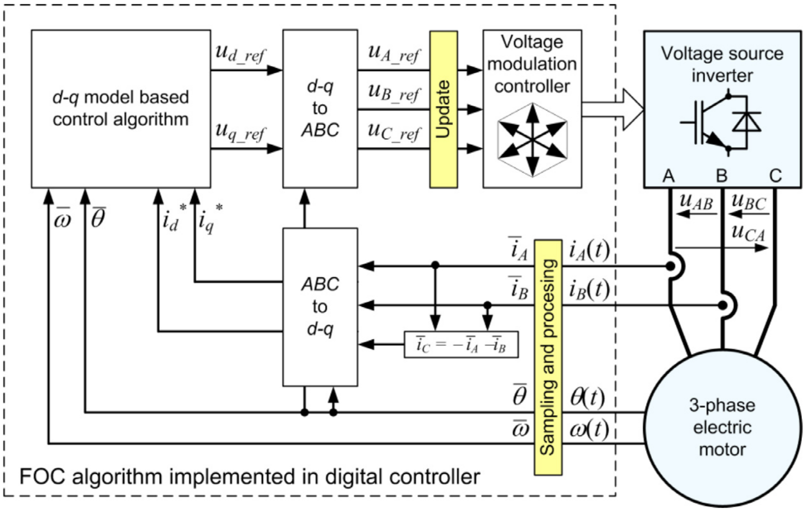

Fig. 1. General structure of FOC applied to PMSM drive.

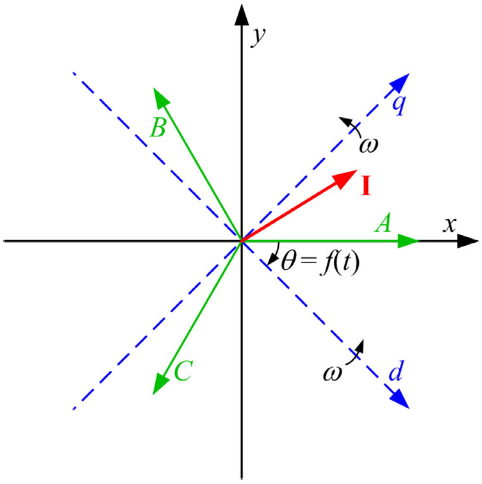

Fig. 2. Reference frames used for modeling and control of PMSM drives.

fundamental control concerns torque, which is related to motor currents in three-phase windings fixed to the stator. As the currents can be regulated rapidly and precisely, the same applies to the torque. Rotor speed and position are controlled by shaping a motor's torque in time, and thus the control of all the mechanical variables is strictly related to regulating motor currents.

Controlling currents require the means to apply variable voltages to the motor terminals and to measure the actual motor currents. Hence, the drives consist of a voltage source inverter and currents' feedback (Fig. 1). The inverter supplies the motor with voltages that have values set by the control algorithm for subsequent discrete control intervals. The currents' feedback enables the algorithm to control the voltages in the closed-loop manner. Typically, two out of three currents i_A, i_B, i_C are physically measured. The third one is computed assuming that the sum of the currents is zero.

Under steady operating conditions, the motor phase currents i_A, i_B, i_C are quasi-sine wave shaped, but during control transients their waveforms are complex and cannot be parameterized by amplitude and frequency. Thus, the drives dedicated to dynamic control use vector control approach [4], which represents motor phase currents by a vector with a modulus and angle that can change rapidly to follow transients. This current vector can be expressed as $\mathbf{I}(i_A, i_B, i_C)$ by using *ABC* coordinates that correspond to the physical layout of motor windings *A*, *B*, and *C*. In order to reduce the number of variables required to define the vector, an *x*–*y* reference frame, as shown in Fig. 2, is commonly used to describe motor currents with two scalar variables i_x and i_y.

In stationary reference frames *ABC* and *x*–*y*, the relation between the motor's torque and currents is complex and involves a rotor position that advances while the rotor revolves. Hence, for control purposes, it is convenient to use a rotating reference frame marked *d*-*q*, with a *d*-axis fixed to the rotor's magnetic field. Such an approach is called field-oriented con-

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