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Quantum inspired evolutionary algorithm for ordering problems

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ABSTRACT

This paper proposes a new quantum-inspired evolutionary algorithm for solving ordering problems. Quantum-inspired evolutionary algorithms based on binary and real representations have been previously developed to solve combinatorial and numerical optimization problems, providing better results than classical genetic algorithms with less computational effort. However, for ordering problems, order-based genetic algorithms are more suitable than those with binary and real representations. This is because specialized crossover and mutation processes are employed to always generate feasible solutions. Therefore, this work proposes a new quantum-inspired evolutionary algorithm especially devised for ordering problems (QIEA-O). Two versions of the algorithm have been proposed. The so-called pure version generates solutions by using the proposed procedure alone. The hybrid approach, on the other hand, combines the pure version with a traditional order-based genetic algorithm. The proposed quantum-inspired order-based evolutionary algorithms have been evaluated for two well-known benchmark applications – the traveling salesman problem (TSP) and the vehicle routing problem (VRP) – as well as in a real problem of line scheduling. Numerical results were obtained for ten cases (7 VRP and 3 TSP) with sizes ranging from 33 to 101 stops and 1 to 10 vehicles, where the proposed quantum-inspired order-based genetic algorithm has outperformed a traditional order-based genetic algorithm in most experiments.

1. Introduction

Ordering problems are extremely broad and relate to many practical applications: allocation of crews in air routes, reverse logistics, allocation of tasks in parallel processing, vehicle routing and production planning (Gopalakrishnan, & Johnson, 2005; Özbakir, & Tapkan, 2011; Sbihi, & Eglese, 2007).

The most well-known of such applications is possibly the travelling salesman problem (TSP) (Applegate, Bixby, Chvatal, & Cook, 2006; Falcone, Chen, & Hamad, 2013), which can be described as follows. Given n locations and assuming that the salesman is in location 1, what should be the sequence of visits, ensuring that each city is visited exactly once, that minimizes the total distance traveled, including the return to location 1? It can be solved through diverse techniques: Branch & Cut (Padberg, & Rinaldi, 1991), Lagrangean Relaxation (Herrero, Ramos, & Guimarans, 2010), Genetic Algorithms (Rani, & Kumar, 2014; Roy, Panja, & Sardar, 2014), Ant colonies (Dorigo, & Gambardella, 1997), Particle Swarm (Kennedy, & Eberhart, 1995), Simulated Annealing (Bayran, & Sahin, 2013), Tabu Search (Malek, Guruswamy, Pandya, & Owens, 1989) and Lin-Kernighan heuristic (Karapetyan, & Gutin, 2011).

Another well-known ordering problem is Vehicle Routing (VRP), where every customer in a certain geographic region has a known demand for products. The demand of each customer must be met in a single delivery. Delivery vehicles start their journey from a warehouse, to which they must return after completing their deliveries. The goal is to determine which vehicle should serve each client so that the total distance traveled by all vehicles is minimized. In addition, the allocation of vehicles to customers must take into account the maximum capacity of each vehicle.

The TSP can be considered a particular case of the VRP with only one vehicle whose capacity is equal to the sum of all customers' demands. Thus, a variety of techniques that solve the VRP can also be applied to the TSP, such as: Branch & Cut (Fukasawa et al., 2006), Heuristics (Pichpibul, & Kawtummachai, 2013) and Evolutionary Computation techniques, such as Genetic Algorithms (Nazif, & Lee, 2012; Prins, 2004), Ant Colonies (Bin, Zhong-Zhen, & Baozhen, 2009), Particle Swarm (Marinakis, & Marinaki, 2010), Simulated Annealing (Breedam, 1995) and Tabu Search (Gendreau, Hertz, & Laporte, 1994).

More recently, a new class of evolutionary computation algorithms, inspired by quantum computing principles, has been developed to achieve better performance in computationally

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intensive problems, called *quantum-inspired evolutionary algorithms*, (Cruz, Vellasco, & Pacheco, 2008; Dias, & Pacheco, 2012; Han, & Kim, 2000, 2002; Pinho, Vellasco, & Cruz, 2009; Vellasco, Cruz, & Pinho, 2010). In some applications, time taken in the calculation of the evaluation function may be critical. In those cases, reaching good solutions with the smallest possible number of evaluations is a very important factor. These quantum-inspired evolutionary models provide good (or optimal) solutions with a smaller number of evaluations, being adequate to solve optimization problems where the evaluation of each possible solution is computationally expensive. These algorithms have been previously used to solve combinatorial and numerical optimization problems, based on binary (Han, & Kim, 2002; Pinho et al., 2009) and real representations (Cruz et al., 2008; Pinho et al., 2009; Vellasco et al., 2010), respectively, providing better results than classical genetic algorithms, with less computational effort. However, for ordering problems, order-based genetic algorithms are more suitable than those with binary and real representations. This is because specialized crossover and mutation processes are employed to always generate feasible solutions.

This work proposes a new quantum-inspired evolutionary algorithm for solving ordering problems with a reduced number of evaluations. The so-called Quantum-Inspired Order-Based Evolutionary Algorithm (QIEA-O) was developed to solve any ordering problem (see Section 2.1), irrespective of the way the cost (evaluation) function is computed and of the elements in the universe, considering permutation or repetition. Quantum-inspired approaches have already been used in ordering problems. However, as discussed in Section 2.2, those approaches are application specific, making use of quantum bits and rotation matrices to solve specific ordering problems, such as the traveling salesman or the knapsack problem. The representation proposed in this paper makes QIEA-O model application independent, allowing it to be applied to any ordering problem.

According to Mark Moore (Moore, & Narayanan, 1995) and Ajit Narayanan (Narayanan, & Moore, 1996), an algorithm, to be considered as quantum inspired, must have the following characteristics:

1. The problem must have a numeric representation, or a method for converting it into a numerical representation should be employed;
2. An initial configuration must be determined;
3. A stopping criterion must be defined;
4. The problem should be able to be divided into smaller subproblems;
5. The number of universes (or superposition states) must be identified;
6. Each problem must be associated with one of the universes;
7. The calculations in different universes must occur independently;
8. Some form of interaction between multiple universes must exist. This interference should allow finding a solution to the problem or provide information for each sub-problem in each universe to be able to find it.

A note should be made with respect to the concept of the universe mentioned above. In quantum mechanics, a particle (an electron, for example) does not have a particular position until it is effectively measured. It is considered that there is a probability distribution for the possible positions of the electron everywhere in space. In other words, the electron behaves as if it could be in infinite locations at the same time. One interpretation of this phenomenon is that the electron exists in several universes simultaneously, and its position is only determined when a measurement is made and the collapse of the probability occurs (the electron ceases to exist in other universes) (Moore, & Narayanan, 1995).

An alternative definition for an algorithm to be considered quantum inspired is presented in Zhang (2010). According to this author, a QIEA (Quantum Inspired Evolutionary Algorithm) is characterized as an evolutionary algorithm where quantum individuals are represented by quantum bits (Qbits) and quantum gates (Qgate) are employed to update individuals. Under this definition Zhang (2010), the proposed algorithm, as well as others in the literature (Cruz et al., 2008; Pinho et al., 2009; Vellasco et al., 2010), might not be considered quantum inspired, since it does not use bits or quantum gates.

The algorithm presented in this paper is based on quantum computing principles established by Moore (Moore, & Narayanan, 1995) and Ajit Narayanan (Narayanan, & Moore, 1996), which allow the optimization process to be carried out with a smaller number of evaluations without using rotation matrices.

This paper has four additional sections. The next section describes the main concepts of Quantum-Inspired Order-Based Evolutionary Algorithm. Section 3 presents the proposed QIEA-O model in details. Section 4 presents the evaluation of the proposed model in two classic problems (the TSP and the VRP, with sizes ranging from 33 to 101 stops, and 1 to 10 vehicles) and a real problem of line scheduling. Finally, Section 5 concludes the work.

2. Background

2.1. Ordering problems

The mathematical formulation of the ordering problem under consideration in this work is:

Minimize $f(\boldsymbol{x})$
Subject to: $\boldsymbol{x} \in X = \{(x_1,\ x_2,\ \ldots.,x_n),$ such that $x_i \in$ U, $\forall i \in \{1,\ 2,\ldots,n\}$ and U finite$\}$
where, f: X → R
$\boldsymbol{x} \rightarrow f(\boldsymbol{x})$

There are two possibilities about the admissible values for a solution:

(1) The solutions $\boldsymbol{x}$ do not allow repetitions of the elements in set U. This is the ordering permutation problem. In this case, the values of n must be equal to m. The TSP is an example of this kind of problem.
(2) The solutions $\boldsymbol{x}$ do allow repetitions of the elements in set U. This is the ordering repetition problem. Notice that in this case the values of n and m have no relationship to each other, although in the most situations $n \geq m$. The production planning is an example of this kind of problem.

2.2. Quantum inspired evolutionary algorithms

Algorithms based on evolutionary computation generate a sequence of solutions along iterations (called generations) so that the value of a criterion function gradually improves.

In genetic algorithms with binary representation, each gene of an individual is represented by a bit. In the case of quantum-inspired genetic algorithms with binary representation, each gene of an individual is represented by a quantum bit.

One quantum bit (Qbit) is the smallest amount of information stored in a quantum computer. Differently from the bit of conventional computing, the value of the Qbit will not always be the same; this value will depend on its observation. In other words, it is not possible to determine a priori the measured value (state) of a Qbit; it is only possible to estimate the probability of observing value 0 or value 1.

Making use of the Dirac notation, a Qbit is represented by: $|\psi = \alpha|0\rangle + \beta|1\rangle$, where $\alpha,\ \beta\ \in$ C, $|\alpha|^2 + |\beta|^2 = 1$, $|\alpha|^2$ and $|\beta|^2$ are