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Optimal prize allocation in contests: The role of negative prizes ☆

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Abstract

In this paper, we analyze the role of negative prizes in contest design with a fixed budget, risk-neutral contestants, and independent private abilities. The effort-maximizing prize allocation rule features a threshold. When the highest effort is above the threshold, all contestants with lower efforts receive negative prizes. These negative prizes are used to augment the prize to the contestant with the highest effort, which better incentivizes contestants with higher abilities. When no contestant's effort exceeds the threshold, all contestants equally split the initial budget (or a portion of it) to ensure their participation. We find that allowing

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negative prizes can increase the expected total effort dramatically. In particular, if no bound is imposed on negative prizes, the expected total effort can be arbitrarily close to the highest possible effort inducible when all contestants have the maximum ability with certainty. The above contest is shown to be the optimal mechanism for a more general class of mechanisms.

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1. Introduction

Many activities and events, such as promotions within organizations, school admissions, political elections, R&D races, and sports, can be viewed as contests. As a well-established institution, contests essentially incentivize contestants to exert costly and irreversible effort through awarding prizes to winners according to their relative rankings. It is often the case that contestants have private information about their own abilities, valuations, competence, etc. A contest designer desires an optimal prize architecture in order to deal with such private information to achieve her objective. In this paper, the objective is to maximize the total effort.[1]

To illustrate the problem we analyze in this paper, imagine that a contest designer has a million dollars to distribute among some ex ante symmetric, but privately informed, risk-neutral workers to incentivize them to exert effort in an all-pay auction. In this environment, Moldovanu and Sela (2001) show that when restricting to fixed nonnegative prizes, the optimal prize structure to maximize the expected total effort is to award the entire prize to the worker with the highest effort. Their result can be understood from the mechanism design intuition: The designer should rank workers according to their "virtual abilities," and the one with the highest virtual ability—who happens to be the one with the highest ability—should be awarded the entire prize (under some regularity assumption), assuming symmetry in workers' ability distributions.

One interesting question is to what extent the insight from Moldovanu and Sela (2001) would generalize if we allow negative prizes to contestants. The intuition to maximize the reward to the highest ability worker in the optimal mechanism is that a dollar can induce more effort from a high-ability worker than from a low-ability worker, since exerting effort is more costly for a lower ability worker. Therefore, the contest designer would like to not only award her entire prize budget to the worker with the highest ability, but also transfer money from those lower ability workers to the highest ability worker if she is allowed to do so. If such monetary cross-type transfers do happen, a lower ability worker may end up with a negative prize ex post. The challenge is then to provide enough incentives for a low-ability worker to participate in the contest. One potential solution is to award every worker if all are of low abilities. As a result, although a low-ability worker may obtain a negative prize with some probability, he obtains a positive prize with the rest of the probability and his interim participation constraint can be satisfied. In Moldovanu and Sela (2001), monetary cross-type transfers involving negative prizes are not allowed, which implies that the total effort elicited from their model is likely to be lower if negative prizes are allowed.

[1] This objective function has been used in many papers in the contest literature; for example, Moldovanu and Sela (2001).