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FORM reliability analysis using a parallel evolutionary algorithm

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ABSTRACT

This paper presents a parallel evolutionary algorithm to solve reliability problems with accuracy and repeatability of results. The last characteristic is usually overlooked; however, it is critical to the reliability of the calculation method itself. Note that evolutionary algorithms are stochastic processes and may not always generate identical results. The optimisation problem resulting from the first order reliability method is considered with an implicit state function that can include a call to a finite element analysis (FEA). A strategy to handle failures from the transformation of random variables or from the finite element call during the evolution process is explained in detail. Several benchmark tests are studied, including some involving bounded random variables that introduce strong non-linearities in the mapping to standard Gaussian space. In addition, the solutions of 2D and 3D frame problems using the finite element method illustrate the capabilities of the algorithm including the convenience of the algorithm in handling discrete limit state functions. Finally, the ability to obtain similar results after many runs is demonstrated.

1. Introduction

The evaluation of reliability in engineering has indeed secured its place in the design and risk analysis of structures. This happened especially after the dissemination of the concept of risk-based design which has been adopted in a number of codes and standards; see e.g. discussion in [1,2]. This situation is a natural consequence of materials, loads and any other external agents being always uncertain, even within a very small margin. In other words, perfection in manufacturing or exactness in loads quantification is only achievable in a stochastic manner and hence uncertainty factors should accompany every specification.

Among popular methods to assess reliability, the first and second order reliability methods (FORM and SORM) became good options due to their simplicity and efficiency. Other alternatives include Monte Carlo simulations that require a greater number of calculations. It is also known that FORM and SORM are able to produce reasonable results in terms of accuracy. Therefore, their acceptance and use have grown widely, particularly in the recent years. This is in part thanks to the improvement of numerical methods to approximate the solution of mechanical problems and the increase of computer power to allow faster and larger computations.

This paper considers the time-invariant first order reliability method (FORM) only [3–7]. In this method, a constrained optimisation problem is deduced for which several techniques have been developed along the years to obtain its solution. This leads to the quantification of the so-called reliability index β. Iterative solutions [8–13] and stochastic ones [14–21] are available but are still being proposed from time to time since challenges yet exist. For instance, derivatives cannot be easily computed in large complex problems involving other numerical methods such as the finite difference or finite element; these derivatives may not be even defined. Moreover, the satisfaction of constraints results in dealing with computer numeric precision which in turn poses difficulties to the solution algorithm. This paper thus presents an approach based on an evolutionary algorithm aiming at solving FORM with accuracy, efficiency and robustness.

Evolutionary algorithms (EA) are based on a progressive update of an initial set of trial solutions (population) by recombining these solutions in a stochastic manner. The original algorithms were inspired by natural selection and evolution where the recombination is made by mimicking genetics; see e.g. [22–25]. There are actually several EAs and related analogies to solve optimisation problems. In addition to overcoming challenges with local optima, EAs have the advantageous capability of generating multiple solutions at the same time. This feature is particularly essential to multimodal problems. One subset of EAs with a promising applicability in reliability analysis is the so-called differential evolution (DE) [26–28]. DEs uses vector differences to recombine solutions instead of genetic analogs.

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It was shown in [29] that the adaptation of differential evolution in the framework of an evolutionary algorithm based on tournaments results in a very efficient algorithm. This algorithm is able, for instance, to produce the same results after several runs; i.e. with a good repeatability behaviour. Note that this feature is not guaranteed by EAs because of their stochastic nature. In addition, the algorithm is able to obtain exact solutions for some particular multi-constrained and multi-objective optimisation problems. The algorithm and related computer code, named Goga, has been made available on the web at https://github.com/cpmech/goga as open (free) software.

In this paper, the aforementioned evolutionary algorithm (Goga) is applied to the solution of the optimisation problem in FORM. In particular, the handling of constraints and the strategy to deal with failures during the computation of the objective value are described in detail. The failures may happen due to the transformation of random variables or due to the inability of a solver such as the finite element method to solve the underlying mechanical problem; for instance when the input (random) parameters are invalid. Several solutions to existent reliability problems having a reference solution are performed in order to investigate the repeatability characteristics of Goga in addition to assess its accuracy. Some challenging reliability problems [30] involving bounded random variables are analysed and 2D/3D frames are studied. It is shown that the EA is fairly robust in dealing with these non-linear problems and with discrete limit state functions.

The paper is organised as follows. A very brief review of FORM is given in Section 2 followed by the description of the evolutionary algorithm and its specialisation to FORM in Section 3. In Section 4, the numerical examples are presented and in Section 5 some conclusions are drawn.

2. Probabilistic analysis and reliability

Reliability is a measure of satisfactory performance of a system (or component) with uncertain characteristics; e.g. geometry, properties, boundary conditions, etc. Therefore, reliability is the opposite of probability of failure. In fact, the probability of failure is a more general quantity that can be readily related to risk when an evaluation of consequences is available. Failure in this context is a generic word that can mean mechanical breakage, inadequate serviceability, or infeasibility, for example.

The *component* reliability is considered in the following. For details on system reliability, the reader is directed to the cited references; e.g. [7].

In a simplified manner, the probability of failure of a component described by two indicators R (resistance or supply) and L (load or demand) can be written as

$$p_f = PR\,[\text{failure}] = PR[R < L] \tag{1}$$

The paradigm resistance-load is illustrated in Fig. 1 where the area under the intersection between the two curves is related to the probability of failure; hence safety cannot be simply expressed by the differences between the mean values as in deterministic approaches.

The difference $(R - L)$ is known as the *limit state function* (LSF); clearly, negative values indicate failure. A more general LSF can be represented by $g(\boldsymbol{x})$ where $\boldsymbol{x}$ is the vector of n random variables: $\boldsymbol{x} = \{x_0, x_1, \ldots, x_{n-1}\}$ (Fig. 2). The hyper-surface $g(\boldsymbol{x}) = 0$ thus delimits the regions of values leading to failure from the regions of values representing a safe situation. The probability of failure in this case can be calculated by means of

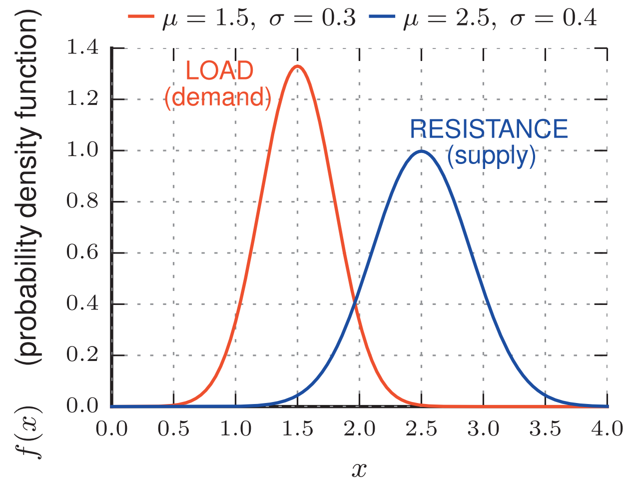

Fig. 1. Resistance-load paradigm in probabilistic analysis.

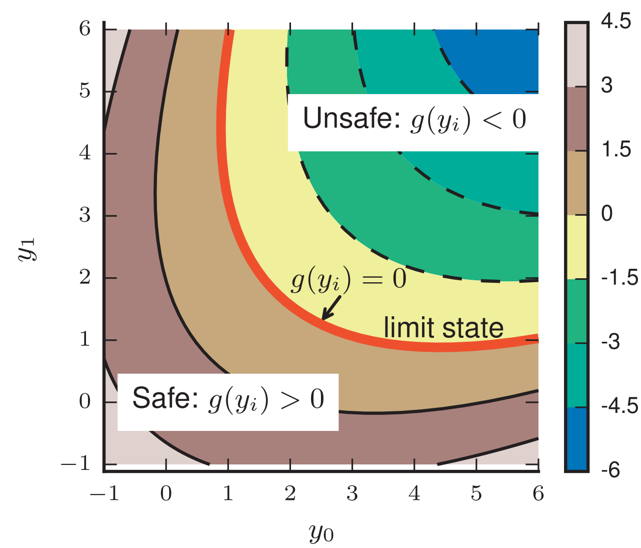

Fig. 2. Limit state function.

$$p_f = \int_{g(\boldsymbol{x}) \le 0} f_x(\boldsymbol{x})\,\mathrm{d}\boldsymbol{x} \tag{2}$$

where f_x is the joint probability density function (PDF) of $\boldsymbol{x}$. The above integral is often difficult to be obtained by analytical means; hence the FORM approximation procedure has been developed. In this method, first, a transformation $\mathbf{T}$ is introduced to convert the space of $\boldsymbol{x}$ into an uncorrelated normal space $\boldsymbol{y}$; this can be symbolised as

$$\boldsymbol{y}(\boldsymbol{x}) = \boldsymbol{T}(\boldsymbol{x}) \tag{3}$$

where the components of $\boldsymbol{y}$ are statistically independent normal variables with zero means and unit standard deviations; see e.g. [31,32]. The cases of correlated and uncorrelated variables are implicitly represented by $\boldsymbol{T}$. Further, the transformation considered in this paper is a direct mapping to the standard Gaussian space. For uncorrelated variables,

$$y_i = \Phi^{-1}(F_{x_i}(x_i)) \tag{4}$$