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An evolutionary nested sampling algorithm for Bayesian model updating and model selection using modal measurement

Feng Qian *, Wei Zheng

Department of Civil and Environmental Engineering, Jackson State University, 1400 John R. Lynch St, Jackson, MS 39217, USA

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ABSTRACT

Nested sampling (NS) is a highly efficient and easily implemented sampling algorithm that has been successfully incorporated into Bayesian inference for model updating and model selection. The key step of this algorithm lies in proposing a new sample in each step that has a higher likelihood to replace the sample that has the lowest likelihood evaluated in the previous iteration. This process, also regarded as a constrained sampling step, has significant impact on the algorithm efficiency. This paper presents an evolutionary nested sampling (ENS) algorithm to promote the proposal of effective samples for Bayesian model updating and model selection by introducing evolutionary operators into standard NS. Instead of randomly drawing new samples from prior space, ENS algorithm proposes new samples from previously evaluated samples in light of their likelihood values without any evaluation of gradient. The main contribution of the presented algorithm is to greatly improve the sampling speed in the constrained sampling step by use of previous samples. The performances of the proposed ENS algorithm for model updating and model selection are examined through two numerical examples.

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1. Introduction

Model updating techniques can be mainly categorized into two categories: deterministic and probabilistic methods. Deterministic methods usually cast model updating into an optimization problem, in which an objective function based on the discrepancy between model prediction and measured data is defined and minimized for improving the model plausibility. Optimization algorithms, including genetic algorithm [1,2], particle swarm optimization [3], simulated annealing algorithm and their derivatives[4], have been applied to model updating. However, deterministic methods can only give one optimal solution, while model updating, like most ill-posed inverse problems, may have more than one potential solution due to insufficient observations, contaminated data or lack of prior knowledge [5,6].

Bayesian probabilistic inference has been demonstrated to be relatively promising in handling practical difficulties of model updating problems [5]. One challenge in implementing Bayesian probabilistic framework is to evaluate the multidimensional integrals over unknown parameter space, which is analytically intractable in general [7]. Thus, stochastic simulation such as importance sampling [8], Gibbs sampling [9], Markov Chain Monte Carlo (MCMC) [10] and their derivative algorithms [11–13] are proposed to steer away from the computation of complexity integrals. However, most sampling methods fail to obtain model evidence that is critical to the evaluation of model plausibility, and are unable to efficiently propose samples in the high probability of posterior distribution for high dimensional problems.

Transitional markov chain monte carlo (TMCMC) algorithm proposed by Ching et al. [11] has been proven to outperform other approaches for its capability of estimating model evidence and smooth convergence merit. However, it has potential problems in tackling higher dimension parameters [12], since its intermediate stage number will augment for the need of more samples while the accuracy of estimators may decrease with increasing parameter dimension [11]. Hybrid Monte Carlo (HMC) method is capable of effectively solving higher-dimensional problems by the guidance of gradient of potential energy [12–13]. Nevertheless, numerically estimating the gradient over all unknown parameters for large complex system is computationally expensive and inaccurate, even infeasible in problems involving a large sample size or streaming data [14]. Furthermore, the leapfrog algorithm involved in the evaluation of Hamilton equations does not always conserve energy in the system with large step size [15], leading to instability of the simulated Hamilton dynamic system, and thereby causing the chain of samples to be trapped in some unexpected regions.

In comparison, NS developed by Skilling [16] is a highly efficient and more convenient sampling algorithm. NS primarily aims to evaluate Bayesian evidence by converting the high-dimensional

* Corresponding author.
E-mail address: qianfeng543@163.com (F. Qian).

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integral into an easily evaluated one-dimensional integral [15,16]. As a by-product, the final set of samples of NS algorithm can be further used to estimate posterior distribution. It has been successfully applied to cosmological field [17], finite element model updating and model selection [18], and surface flow problems [15,19]. There is still an increasing demand for NS algorithm to solve statistical inference and Bayesian model selection for its easy and convenient implementation. The main problem of the standard NS algorithm lies in the constrained step in which a new sample with higher likelihood is proposed to replace the sample with the lowest likelihood. This issue becomes even more challengeable after several iterations as the likelihood has reached a higher value and the parameter space shrinks to a very sharp region. To address this problem, Elsheikh [15] has incorporated the HMC into standard NS algorithm. However, the estimation of energy gradient required in HMC method is computationally expensive and inaccurate as aforementioned. Therefore, more efficient and feasible methods are still needed to be explored for promoting standard NS algorithm by effectively proposing valid samples.

This paper presents an ENS algorithm for Bayesian model updating and model selection. ENS is a combination of evolutionary algorithm (EA) and standard NS method. Instead of randomly proposing new samples from prior distribution in the standard NS, ENS generates new samples from previous samples by evolutionary operations, e.g. selection, crossover and mutation. The samples evaluated in the former steps are ranked based on their likelihood values and probably selected to evolve into higher region of posterior distribution. The advantage of the proposed ENS algorithm over standard NS is to make full use of previous samples to draw new samples without any evaluations of gradient. Furthermore, numerical results show that ENS has the capacity of dealing with multimodal problems with sufficient initial samples. The sampling efficiencies of ENS and standard NS are compared. The performance of the presented ENS algorithm for model updating and model selection is examined based on two numerical examples, e.g. a clamped-clamped beam and a truss structure.

2. Theory of finite element model updating

The essence of finite model updating is to determine or calibrate the unknown structural parameters to make the predictions of a numerical model match with field measurements as much as possible. Such parameter calibration process can be realized by minimizing the difference between the field measurement and the prediction of the theoretical model. In practice, structural modal characteristics (e.g. modal frequencies and mode shapes) can be identified from time history responses through stochastic subspace identification [20], Hilbert-Huang transform [21] and empirical mode decomposition methods [22]. Let $D = \{\hat{\omega}_r, \hat{\phi}_r^k \in R^{N_0}, r = 1,\ldots,m, k = 1,\ldots,N_d\}$ denote the extracted modal data at the selected N_0 observation DOFs, where $\hat{\omega}_r$ and $\hat{\phi}_r^k$ are the r-the circular natural frequency and mode shape, N_d is the number of data sets available, and m refers to the number of observed modes.

Suppose $\boldsymbol{\theta} = \{\theta_0, \theta_1, \theta_2, \ldots, \theta_\tau\}$ is the unknown parameter vector characterizing the theoretical model of a real structure, where τ is the number of unknown structural parameters. The prediction of the parameterized model can be obtained from its eigenvalue equation and denoted as $\{\omega_r(\theta), \phi_r(\theta) \in R^{N_0}, r = 1,\ldots,m\}$. The difference between measured modal data D and model prediction for the r-the modal frequency and mode shape component can be respectively defined as [23,24],

$$J_{\omega r}(\theta) = \frac{1}{N_D}\sum_{k=1}^{N_D}\frac{[\omega_r(\theta) - \hat{\omega}_r^k]^2}{[\omega_r^k]^2} \text{ and } J_{\varphi r}(\theta) = \frac{1}{N_D}\sum_{k=1}^{N_D}\frac{\|\beta_r^k\varphi_r(\theta) - \hat{\phi}_r^k\|^2}{\|\hat{\phi}_r^k\|_{N_0}^2} \quad (1)$$

where $\|\cdot\|^2$ means Euclidian norm, and $\|\cdot\|_{N_0}^2 = \|\cdot\|^2/N_0$. $\beta_r^k = \hat{\phi}_r^k\phi_r(\theta)/\|\phi_r(\theta)\|^2$ is a normalized factor that ensures the measured mode shape $\hat{\phi}_r^k$ closest to the mode shape $\beta_r^k\phi_r(\theta)$. $J_{\omega r}(\theta)$ and $J_{\phi r}(\theta)$ in Eq. (1) respectively give the mean errors of the r-th frequency and mode shape between the measured modal data and model predictions. Thus the overall error can be obtained from the weighted combination of the contribution of all m modal frequencies and mode shapes.

$$J(\theta) = \sum_{r=1}^{m} J_{\omega r}(\theta) + w\sum_{r=1}^{m} J_{\phi r}(\theta) \quad (2)$$

where w is the weighting factor that reflects the relative importance of error contributed by mode shapes. Goller and Beck et al. [25] studied the optimum selection of the weighting factors in model updating from the view of Bayesian model evidence.

3. Bayesian model updating and model selection

The main idea of applying Bayesian approach to model updating is to obtain a posterior probability density function (PDF) $p(\boldsymbol{\theta}|D, M_j)$ of uncertainty parameters $\boldsymbol{\theta}$ for the model class M_j based on available measurement data D. The model class refers to a group of different models resulted from different possible model parameters $\boldsymbol{\theta}$ due to the associated uncertainties. The key step is to establish the likelihood function that describes the probability of obtaining the measurement D based on the model defined by $\boldsymbol{\theta}$. The error between model prediction and measurement D is usually assumed to be a zero-mean stationary normally-distributed stochastic process with a shared standard deviation σ. Thus, the likelihood function can be expressed as the conjunct normal distribution of multiple independent variables as following:

$$p(D|\theta, M_j) = \frac{1}{(\sqrt{2\pi}\sigma)^{m(N_0+1)}}\exp\left(-\frac{J(\theta)}{2\sigma^2}\right) \quad (3)$$

The shared standard deviation σ is also unknown and needs to be determined, extending the unknown parameter vector to be $\mathbf{x} = \{\theta_0, \theta_1, \theta_2, \ldots, \theta_m, \sigma\}^T = \{\theta^T, \sigma\}^T$.

Thus, the posterior probability density function (PDF) of the uncertainty parameters can be obtained as following according to Bayesian theorem [5],

$$p(\mathbf{x}|D, M_j) = \frac{p(D|\mathbf{x}, M_j)p(\mathbf{x}|M_j)}{p(D|M_j)} \quad (4)$$

where $p(D|\mathbf{x}, M_j)$, $p(\mathbf{x}|M_j)$ and $p(D|M_j)$ are likelihood function, prior PDF and model evidence, respectively.

The model evidence $p(D|M_j)$ is the marginal likelihood function that describes the quality of the model specified by unknown parameters. It can be determined as below,

$$\begin{aligned} p(D|M_j) &= \int p(D|\mathbf{x}, M_j)\cdot p(\mathbf{x}|M_j)d\mathbf{x} \\ &= \int\left(\frac{1}{(\sqrt{2\pi}\sigma)^{m(N_0+1)}}\cdot\exp\left\{-\frac{J(\theta)}{2\sigma^2}\right\}\right)\cdot p(\mathbf{x}|M_j)d\mathbf{x} \end{aligned} \quad (5)$$

Commonly, the multidimensional integral (summation for discrete model) over all the space of unknown parameter vector $\mathbf{x}$ in the Eq. (5) cannot be analytically evaluated due to its higher dimension. Thus, ENS method is elaborated in this paper to estimate the model evidence in Eq. (5) and approximate the posterior PDF of the uncertainty model parameters in Eq. (4) with the byproduct samples.

Because uncertainties associated with model classes, there may be many other competitive model classes that could more accurately represent the real system based on measurements. The Bayesian model selection then can be adopted to determine the most plausible model by their corresponding model probabilities