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# Statistical properties of user activity fluctuations in virtual worlds

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ABSTRACT

User activity fluctuations reflect the performance of online society. We investigate the statistical properties of 1 min user activity time series of simultaneously online users inhabited in 95 independent virtual worlds. The number of online users exhibits clear intraday and weekly patterns due to human's circadian rhythms and weekly cycles. Statistical analysis shows that the distribution of absolute activity fluctuations has a power-law tail for 44 virtual worlds with an average tail exponent close to 2.15. The partition function approach unveils that the absolute activity fluctuations possess multifractal features for all the 95 virtual worlds. For the sample of 44 virtual worlds with power-law tailed distributions of the absolute activity fluctuations, the width of singularity $\Delta\alpha$ is negatively correlated with the maximum activity ($p$-value = 0.070) and the time to the maximum activity ($p$-value = 0.010). The negative correlations are not observed for neither the other 51 virtual worlds nor the whole sample of the 95 virtual worlds. In addition, numerical experiments indicate that both temporal structure and large fluctuations have influence on the multifractal spectrum. We also find that the temporal structure has a stronger impact on the singularity width than large fluctuations.



## 1. Introduction

A massive multiplayer online role-playing game (MMORPG) forms an online virtual world, where people can work and interact with one another in a somewhat realistic manner. Therefore, virtual worlds have great potential for research in the social, behavioral, and economic sciences [1]. For instance, we can embed evolutionary games in a virtual world to study the formation of human cooperation [2] and to understand the evolution of wealth distribution [3]. A pioneering work was done by Castronova, who traveled in a virtual world called "Norrath" and performed preliminary analysis of its economy [4]. Recently, there have been also efforts in the field of computational social sciences from a complex network perspective [5–11]. In addition to its scientific potentials, virtual worlds could act as nice places for real social activities, such as marketing [12–14], and provide opportunities for players to make real money [15].

The number of instant online users is an important indicator for scientific and commercial purposes. The number of registered users is closely related to the profit of an MMORPG company and the instant number of online users shows the degree of popularity of an MMORPG [16]. The number of instant online users is an analogue to various instant society flows [17,18]. Moreover, we note that the online-offline activities of users have the power to identify game cheaters and the gaming session durations of the majority of normal users are distributed according to the Weibull distribution [19], which deviates the power-law bursts of human activities in many social systems [20]. In addition, power-law behavior extensively exists in social and natural sciences [21,22], which is identified in our investigated data. In a word, it is meaningful to study the linear and nonlinear dynamics of the number of instant online users and duration between login and logoff moments. We mainly focus on the multifractal nature of the absolute fluctuations of user activities (the absolute increments of instant numbers of simultaneously online users) in this work.

Multifractals is ubiquitous in natural and social sciences [23]. Many different methods have been applied to characterize the hidden multifractal behavior of different social variables, such as the fluctuation scaling analysis [24,25], the structure function method [26–29], the multifractal detrended fluctuation analysis (MF-DFA) [30–32], the multifractal detrending moving average analysis (MF-DMA) [33], the partition function method [34–38], the multiplier method [39–41], the wavelet transform approaches [42,43], and

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the microcanonical multifractal analysis [44,45], some of which are borrowed from the multifractal analysis of turbulence data. We apply the partition function approach to the absolute fluctuation time series of 1 min online user number to uncover the multifractal nature of the records in the present study.

The rest of this paper is organized as follows. Section 2 describes the data used in our study, including the time series of user activities and its fluctuations. Section 3 investigates the intraday patterns and weekly patters of user activities and Section 4 studies the probability distribution of the fluctuations of user activities $|\Delta N|$. We perform multifractal analysis of the user activity fluctuations based on the partition function approach and unfold the relationships between multifractal nature and the performance of virtual worlds. We summarize our findings in Section 6.

## 2. Data description

We use a huge database recorded from 95 servers of a popular MMORPG in China to uncover the patterns characterizing virtual worlds. Our data set contains all in-game action logs for 111 days from May 16 to September 4 in 2011. However, we mainly focus on the online-offline logs in this study. An entry is written to the log file when a user goes offline. Therefore, the entries in a log file are arranged according to an increasing order of logout moments. Each entry contains three pieces of information: the masked user ID, its login time, and its logout time. The resolution of the time stamps is 1 second. For each user, we collect all the associated entries. During this period, on average, there were more than 100 000 users created on each server. For security sake, the true user IDs have been encrypted into numbers from 1 to the ordinal number of the last ID for each virtual world.

We use 1 min number $N_i(t)$ of simultaneously online users as the user activity of the $i$th virtual world. Considering the privacy of the data, we define a quantity $n_i(t)$ as a substitute for $N_i(t)$, which does not change the results,

$$n_i(t) = N_i(t)/N_{\max}, \tag{1}$$

where

$$N_{\max} = \max_i \{N_{i,\max}, i = 1, 2, \ldots, 95\} \tag{2}$$

in which $N_{i,\max}$ is the maximum of the user activities $N_i(t)$ of the $i$-th virtual world:

$$N_{i,\max} = \max_t \{N_i(t), t = 1, 2, \ldots, T\} \tag{3}$$

Accordingly, the relative maximum 1 min number of online users can be calculated as follow:

$$n_{i,\max} = N_{i,\max}/N_{\max}. \tag{4}$$

We find that the majority of $n_{i,\max}$ are greater than 0.8 and the mean is 0.8105, which indicates that there exist small differences in the maximum activity $N_{i,\max}$ among most virtual worlds. Meanwhile, we have removed the abnormal activities (e.g. when the servers were scheduled for maintaining or during game version updating) of the 95 virtual worlds in order to ensure statistical significance.

Fig. 1 illustrates the evolution of 1 min online user numbers for a typical server during the period under investigation. The maximum relative activity $n_{i,\max}$ is reached at the beginning of the recording period. Especially, there exist two evident local humps in the plot around 2011/05/21 and 2011/07/20. These humps are mainly caused by some new marketing actions organized by the online game operators. We find that other curves almost share the same shape as in Fig. 1 except for some special dates, and the rest time series of the virtual worlds also have similar features.

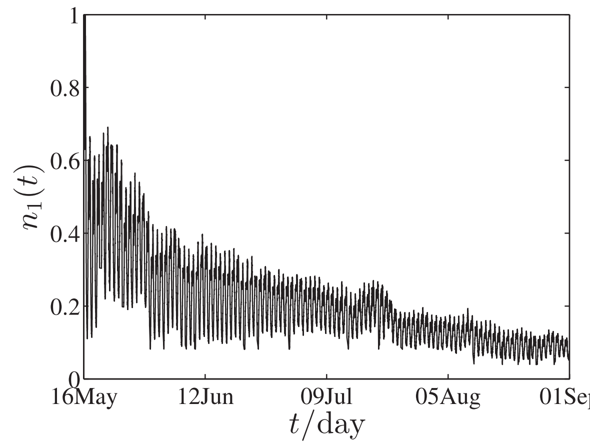


**Fig. 1.** Time series of the relative number $n_1(t)$ of online users minute by minute for a typical virtual world.

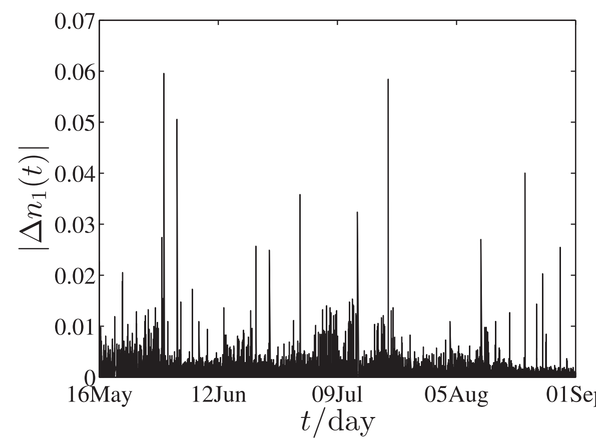


**Fig. 2.** Evolution of the fluctuations $|\Delta n_1(t)|$ of user activities in the same typical virtual world used in Fig. 1.

Fig. 2 illustrates the evolution of absolute fluctuations of online user activities, which is the absolute difference

$$\Delta n_1(t) = |n_1(t) - n_1(t-1)|. \tag{5}$$

One can find that the time series exhibits large fluctuations and intermittent behavior. In addition, Eq. (5) is a substitute of Eq. (6), which does not change the results.

$$\Delta N_1(t) = |N_1(t) - N_1(t-1)|. \tag{6}$$

## 3. Intraday pattern and weekly pattern

In order to investigate the seasonal patterns in the time series of the online user activities, we calculate the average number $A_i(\tilde{t})$ of online users as follows

$$A_i(\tilde{t}) = \frac{1}{M_i} \sum_{j=1}^{M_i} n_i^j(\tilde{t}), \tag{7}$$

where $i = 1, 2, 3, \ldots, 95$ and $M_i$ is the number of operating days in the $i$th virtual world, $n_i^j(\tilde{t})$ is the 1 min relative number of online users, which is divided by its maximum at time $\tilde{t}$ of day $j$ as defined in Eq. (1).

We first determine the intraday patterns on working days respectively for Monday, Tuesday, Wednesday, Thursday and Friday, as presented in Fig. 3. Roughly speaking, the five curves almost overlap and no remarkable differences are observed among these days. On average, the maximum number of online users is reached