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Algorithms for the orientation of a moving object with separation of the integration of fast and slow motions☆

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ABSTRACT

Equations and algorithms for determining the orientation of a moving object in inertial and normal geographic coordinate systems are considered with separation of the integration of the fast and slow motions into ultrafast, fast and slow cycles. Ultrafast cycle algorithms are constructed using a Riccati-type kinematic quaternion equation and the Picard method of successive approximations, and the increments in the integrals of the projections of the absolute angular velocity vector of the object onto the coordinate axes (quasicoordinates) associated with them are used as input information. The fast cycle algorithm realizes the calculation of the classical rotation quaternion of an object on a step of the fast cycle in an inertial system of coordinates. The slow cycle algorithm is used in calculating the orientation quaternion of an object in the normal geographic coordinate system and aircraft angles. Results of modelling different versions of the fast and ultrafast cycle algorithms for calculating the inertial orientation of an object are presented and discussed. The experience of the authors in developing algorithms for determining the orientation of moving objects using a strapdown inertial navigation system is described and results obtained by them earlier in this field are developed and extended.

The authors of this paper have been occupied with the development of the theory and algorithms of strapdown inertial navigational systems (SINS) from the middle of the 1970s.[1–14] Their experience in developing the provision of algorithms for SINS in relation to the construction of on-board algorithms for determining the orientation of moving objects in inertial and normal geographic coordinate systems (NGCS) is extended in this paper. Ultrafast, fast and slow cycles of the orientation algorithms of SINS which, using the on-board computer, carry out the separate integration of the fast and slow angular motions of an object and the (NGCS) using initial integral information from the gyroscopes concerning the rotational motion of the object are also considered. The immediate use of this information makes it impossible to employ classical methods for integrating the differential equations of the inertial orientation of an object and requires the development of special algorithms for integrating these equations. In the ultrafast and fast cycles, the fast absolute motions of the object are integrated in the inertial system of coordinates and, in the slow cycle, the slow angular motions of the NGCS are integrated in the same system of coordinates.

The fast cycle algorithms were constructed using a Riccati-type kinematic quaternion equation (KQE)[7] (also, see Refs 9–14) and the Picard method of successive approximations. The rotation quaternion $\boldsymbol{x}$ with a zero scalar part appears as the variable and a correspondence is established with the classical Hamiltonian rotation quaternion $\boldsymbol{\lambda}$[7,15,16] using a quaternion analogue of Cayley's formula.[7,17] The algorithms are of the second, third and fourth order of accuracy and, using the initial integral information from the gyroscopes regarding the rotational motion of the object, on one step of the ultrafast cycle they form the increment $\delta\boldsymbol{x}$ in the rotation quaternion $\boldsymbol{x}$ in the inertial system of coordinates. To sum the increments $\delta\boldsymbol{x}_1$, $\delta\boldsymbol{x}_2$,..., $\delta\boldsymbol{x}_n$ within the ultrafast cycle (to form the increment $\Delta\boldsymbol{x}$ of the rotation quaternion $\boldsymbol{x}$ on a step of the fast cycle), a new formula for the addition of the finite rotations described by the quaternions $\delta\boldsymbol{x}_i$ or approximations of it of different orders of accuracy[12,14] is used.

The fast cycle algorithm realizes the change from the increment $\Delta\boldsymbol{x}$ in the rotation quaternion $\boldsymbol{x}$ to the increment $\Delta\boldsymbol{\lambda}$ of the classical rotation quaternion on a step of the fast cycle using a quaternion analogue of Cayley's formula[7,17] and also implements the quaternion

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formula for the addition of finite rotations proposed by Branets and Shmyglevskii[7,15,16] for calculating the orientation quaternion of an object $\boldsymbol{\lambda}$ in the inertial system of coordinates at the current instant. Results of the mathematical modelling of different versions of the fast and ultrafast cycle algorithms for calculating the inertial orientation parameters of the object are presented and analysed later.

The slow cycle algorithm is used to calculate the classical orientation quaternion in the NGCS using increments (on a step of the slow cycle) of the classical orientation quaternions and the NGCS in the inertial system of coordinates. The algorithm either implements a quaternion computational scheme in increments or a matrix computational scheme using commutating quaternion matrices of two types.[7,18] The effect of the commutativity of quaternion matrices enables the construction of simpler algorithms for the slow cycle. The quaternion of the inertial orientation of the NGCS is calculated using numerical integration of the KQE describing the rotation of the NGCS by the mean velocity method (with an algorithm of the second order accuracy) using information generated by the SINS.

It has already been noted that the separation of the SINS orientation algorithms into ultrafast, fast and slow cycles makes it possible, using the on-board computer, to accomplish the separate integration of the fast and slow angular motions of an object and the NGCS which allows us to increase the accuracy of the numerical solution of the orientation problem and to reduce the load on the on-board computer as well as to develop compact highly accurate specialized algorithms for the integration of fast motions (that is, the absolute angular motions of an object) that make immediate use of the initial integral information from the gyroscopes concerning the angular motion of the object.

Note that the introduction of ultrafast and fast cycles allows us to compensate for the error in the determination of the inertial orientation of manoeuvrable and highly manoeuvrable aircraft due to conical motions of the base of the SINS caused by high frequency vibrations, angular accelerations and other dynamic loadings and overloadings of the base of the SINS. Conical motions of the base of the inertial sensing elements are a considerable source of errors and noise in contemporary SINS. To compensate for them, it is therefore necessary to calculate the motions in the ultrafast SINS cycle with a frequency that exceeds the frequency of the motions of the base of the SINS by a minimum of 10 to 20 times.

An increase in the accuracy of the determination of the orientation of objects is essential for all SINS. A reduction in the load on the on-board computer is especially urgent for the SINS used in systems for the orientation, navigation and control of the motion of small objects of civilian and military importance (miniature robots, drones, missiles, bombs ("smart" weapons) since the resources of the on-board computers (processors) of the navigation and motion control systems of such objects are quite limited.

These and other reasons that give rise to the need to separate the SINS orientation algorithms into ultrafast, fast and slow cycles are considered in greater detail in Sections 2 and 3.

1. Equations of the problem of determining the orientation of a moving object using a SINS in Euler (Rodrigues–Hamilton) parameters

The operational algorithms of a SINS, that serve to determine the orientation of a moving object in the inertial and normal geographic coordinate systems, enable the on-board calculation of some or other of its orientation parameters using signals from the sensors of the SINS (the projections of the absolute angular velocity vector of the object onto the axes fixed to the object (or the increments in the integrals of these projections)) and information on the geographic latitude and longitude of the location of the object or information on the projections of the absolute angular velocity of the NGCS on its coordinate axes and the altitude and latitude of the object. The construction of these algorithms is based on the equations for the ideal operation of a SINS, that is, on the basis of differential and functional relations connecting the orientation parameters of an object with the quantities measured by the SINS sensors (subject to the condition of their ideal operation) or ideally generated by the SINS. Different versions of the equations for the ideal operation of a SINS are possible[3–7,16,19–24] that differ in the form of the kinematic orientation and navigation parameters used and in the choice of the coordinate systems in which the differential orientation and navigation equations are integrated.

In solving problems of navigation and determining the object orientation using a SINS, it is necessary to integrate the kinematic orientation equations in some form or other and to transform the coordinates by means of some or other kinetic parameters. Euler–Krylov angles, direction cosines and Euler (Rodrigues–Hamilton (RH)) parameters are among the most widely used kinematic parameters in inertial navigation. Inertial orientation and navigation equations and relations in direction cosines and Euler parameters have well known analytical and computational advantages over the equations and relations in Euler–Krylov angles.[1,2,7,15,16,25,26] Euler–Krylov angles are therefore not competitively capable in the majority of problems concerning the orientation and navigation of a moving object solved by means of a SINS when compared with direction cosines and Euler (RH) parameters.

When direction cosines are used in the equations for the ideal operation of a SINS, differential equations for the object orientation and the NGCS in direction cosines are employed that have the form of kinematic Poisson equations. Analysis shows[2,7] that, in the case when an algorithm of first order accuracy (that is, the simplest Euler method that is the first approximation to the mean velocity method[2,7,15,23]) is used to integrate the differential orientation equations, direction cosines have an advantage in the sense of the volume of calculations carried out in one step in the execution of the inertial orientation equations of the SINS. In cases when algorithms of the second or higher order of accuracy are used to integrate the orientation equations, Euler parameters have the advantage in this sense (in particular, this already holds when used in the algorithm for the numerical integration of the second approximation to the mean velocity method).

When integrating the orientation equations in Euler parameters and the similar equations in direction cosines using an algorithm of first order accuracy, the step size in the integration must be small (as a rule, it is thousandths of a second). Numerical integration algorithms of the second and third orders of accuracy allow the step size to be considerably increased (by an order of magnitude or more) and the load on the on-board computer to be reduced. It should also be noted that the numerical integration algorithm in Euler parameters that executes one of the basic methods, that is, the mean velocity method widely used in SINS for cosmic assignments,[23] gives a better accuracy in determining the orientation of an object than the similar algorithm in direction cosines.[2,7] Moreover, in determining the object orientation with respect to the normal system of coordinates (geographic or orthodromic), the use of Euler parameters allows a rational computational scheme in increments that utilize the effect of the commutability of the two types of quaternion matrices to be obtained.[7]

According to the above discussion, the equations for the ideal operation of a SINS, written using Euler (RH) parameters, have, in the majority of cases, an advantage over equations written using Euler–Krylov angles or direction cosines. The equations and algorithms for determining the orientation of a moving object using a SINS are therefore written here in Euler parameters as the basic parameters.