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Algorithms of Fast Adaptive Compensation of Disturbance in Linear Systems with Arbitrary Input Delay§

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Abstract: The problem of fast compensation of multisinusoidal disturbance affecting on linear time-invariant (LTI) plants with known parameters and arbitrary input delay is considered. The disturbance is modeled as unmeasurable bounded output of linear exosystem of known order but with unknown parameters. Disturbance compensation is provided by means of direct adaptive control with two schemes of adaptation algorithm. The first scheme is based on the standard gradient adaptation algorithm, while the second one uses modified adaptation algorithm with integral cost function and improved parametric convergence. The stability of the closed-loop system in presence of delay and integral action (due to direct adaptation) in the feedforward loop is ensured by appropriate modifications of the adaptation schemes. These modifications allow to remove the limitations on upper bounds on adaptation gains (gain margins) corresponding to certain upper bounds of time delays (delay margins). The performance of proposed schemes of disturbance compensation is demonstrated via simulation.

Keywords: Adaptive control, disturbance compensation, delayed systems, adaptation algorithms with improved parametric convergence.

1. INTRODUCTION

The problem of adaptive multisinusoidal disturbance compensation for a class of LTI plants with arbitrary input delay is considered in the paper. The main issue of the problem is in improvement of adaptation rate.

The problem of disturbance compensation was intensively researched during last 30 years and successfully resolved for linear plants (Basturk et al. (2014), Bodson et al. (1997), Marino et al. (2003), Nikiforov (1996)) and different classes of nonlinear systems (Nikiforov (1997), (1998), (2001), (2003) and references therein). Most of the proposed schemes of compensation are based on internal model principle introduced by Francis et al. (1975) and Johnson et al. (1971). The principle implies design of disturbance generator as autonomous linear model, called exosystem, and incorporation of the parameters of this model into control.

Internal model principle has also been successfully used for the problems of disturbance compensation in uncertain systems with delays (Annaswamy et al. (2008), Basturk et al. (2015), Bobtsov et al. (2008), Gerasimov et al. (2015), Pyrkin et al. (2010)), where the principle is applied in the framework of indirect adaptation. The key element in these solutions is a standalone block designed for identification of multisinusoidal disturbance parameters — frequencies, amplitudes, phases.

On the other side, the attempts of direct adaptive control application in presence of input delay are undertaken by Annaswamy et al. (2008) and Gerasimov et al. (2015). The crucial problem in these works, as in direct adaptation in general, is in forced adaptation gain limitation for a certain input delay. The gain is to be chosen below some upper bound called gain margin to ensure the stability of the system. Gain margin corresponds to some upper bound of delay called time delay margin. On the one hand, the need of this limitation is caused by incorporation of integral type adaptation algorithm together with delay in open loop (of the closed-loop system). On the other hand, the limitation leaves less room for performance improvement.

Using results presented in Gerasimov et al. (2016a), Gerasimov et al. (2016b) we propose an adjustable control law compensating uncertain external disturbances affecting on delayed linear plant. The proposed control law uses adaptation algorithm with integral cost function and dramatically improved parametric convergence. The paper presents a general scheme of such an adaptation algorithm as well as its certain implementation for the problem of external disturbance compensation.

The remaining of the paper is organized as follows. In section 2 the problem of disturbance compensation is formulated. In section 3 parameterization of disturbance is

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presented for further controller design. In section 4 two schemes of adaptation algorithm are presented. In section 5 simulation results are demonstrated and compared. In Appendix A general scheme of adaptive algorithm with improved parametric convergence is derived.

2. PROBLEM STATEMENT

Let us consider linear time-invariant plant with the input delay[1]:

$$\dot{x} = Ax + b(u(t-\tau) + \delta), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, u is the control, A is known $n \times n$ matrix, $b \in \mathbb{R}^n$ is known vector, τ is the known constant delay, δ is unmeasurable and bounded external disturbance.

We accept the following assumptions concerning the plant and disturbance.

Assumption 1. Known matrix A is Hurwitz.

This assumption is accepted to focus the main attention on the problem of adaptive disturbance compensation in presence of input delay. Different approaches to the problem of stabilization of linear delayed system can be found in literature (Krstic (2009), Pyrkin et al. (2015), Richard (2003) and references therein). Thus, the main results of the paper can be directly extended for the case of unstable plants.

Assumption 2. External disturbance δ can be presented as the output of linear exosystem

$$\begin{cases} \dot{z} = \Gamma z, \; z(0), \\ \delta = h^T z, \end{cases} \tag{2}$$

where $z \in \mathbb{R}^m$ is unmeasurable state vector, $\Gamma \in \mathbb{R}^{m \times m}$ is a constant matrix with simple eigenvalues on the imaginary axis, $h \in \mathbb{R}^m$ is a constant vector. The pair (h^T, Γ) is assumed to be observable.

Assumption 2 implies that disturbance δ is presented by the class of multisinusoidal signals with bias. This class of signals is widespread in many engineering applications.

Assumption 3. The dimension m of the exosystem (2) is known, but parameters of matrix Γ and vector h are unknown.

Assumption 3 implies that the disturbance δ is *a priori* uncertain.

The objective considered is to design a state-feedback control providing boundedness of all the closed-loop signals and the convergence of state x to zero:

$$\lim_{t \to \infty} \|x(t)\| = 0 . \tag{3}$$

[1] Here and hereafter the argument t is omitted except delayed arguments.

3. DISTURBANCE PARAMETERIZATION AND ADJUSTABLE CONTROL LAW

We use linear parameterization of uncertain disturbance in the form (Nikiforov (1996), (1998), (2003), (2004))

$$\delta(t) = \theta^T \xi + \varsigma, \tag{4}$$

where $\theta \in \mathbb{R}^m$ is the vector of constant unknown parameters depending on Γ and h, ς is the exponentially decaying term caused by nonzero initial conditions, $\xi \in \mathbb{R}^m$ is regressor generated by the filter (*virtual disturbance observer*)

$$\dot{\xi} = G\xi + l\delta \tag{5}$$

with arbitrary Hurwitz matrix G and constant vector l, such that the pair (G, l) is controllable.

Remark 1. The term ς does not influent on stability of the closed-loop system and final result of disturbance compensation. Therefore signal ς will be omitted.

Substitution of (4) into (5) gives the following *canonical* form for the virtual disturbance observer:

$$\dot{\xi} = \left(G + l\theta^T\right)\xi . \tag{6}$$

Since observer (5) includes unmeasurable signal δ, models (5) and (6) are not implementable (that is why they are called *virtual*). However, we can form an estimate $\hat{\xi}$ of the virtual regressor ξ with the use of the following physically implementable *disturbance observer* (DOB) proposed by Nikiforov (2004):

$$\begin{aligned} \hat{\xi} &= \eta + Nx, \\ \dot{\eta} &= G\eta + (GN - NA)x - Nbu(t-\tau), \end{aligned} \tag{7}$$

where $\eta \in \mathbb{R}^m$ is the auxiliary signal, $N \in \mathbb{R}^{m \times n}$ is the arbitrary matrix obeying equality $Nb = l$. By introducing observation error $e_\xi = \xi - \hat{\xi}$ and differentiating it on account of (1), (5) and (7) we prove its exponential convergence to zero. As a consequence, parameterized disturbance (4) can be rewritten in the form of linear regression $\delta = \theta^T \hat{\xi}$ with the state $\hat{\xi}$ generated by physically implementable filter (7).

To prevent the negative influence of input delay we design the predictor of disturbance behavior. The predictor is based on fundamental solution of (6):

$$\xi(t+\tau) = R\xi ,$$

where expression $R = e^{(G + l\theta^T)\tau}$ denotes the matrix exponent. Since the vector θ is unknown, value $\xi(t+\tau)$ is not implementable. Substituting ξ for its estimate $\hat{\xi}$ we obtain:

$$\hat{\xi}(t+\tau) = \hat{R}\hat{\xi} \tag{8}$$

and