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ScienceDirect

Procedia Computer Science 107 (2017) 802 – 807

Procedia Computer Science

International Congress of Information and Communication Technology (ICICT 2017)

Application Research on Sparse Fast Fourier Transform Algorithm in White Gaussian Noise

Liu Zhong*, Li Lichun, Li Huiqi

National Digital Switching System Engineering & Technique R&D Center, Zhengzhou 450001，China
** Corresponding author：liuzhongsky@163.com Tel.:+86-18538565272*

Abstract

In sparse fast Fourier transform algorithm, noise will increase the difficulty in frequency location. As to this problem, probability of detected frequency are analyzed with respect to noise level and bucket in this paper. Firstly different mean and variance of compressed vector in frequency domain are derived under the hypothesis of whether there is a signal, then these statistical characteristics are used to analyze the impact of signal-to-noise ratio and number of point per bucket on detection probability of frequency. Finally, simulation curves is given under the conditions of different noise and bucket. Simulation shows that frequency of signal with additive white Gaussian noise could be effectively detected when SNR is higher than 10dB and number of point per bucket smaller than 2^{12}. And in order to ensure effective detection of frequency, when SNR decrease, number of point per bucket should be reduced. Through the analysis, this paper provides a theoretical support to enhance the reliability of the algorithm.

Keywords: sparse fast Fourier transform; statistical characteristics; sparse signal; additive white Gaussian noise

1. Introduction

In the area of signal processing, discrete Fourier transform (DFT) is one of the most fundamental algorithms. And the FFT is the fastest DFT algorithm, which achieves $O(n\log n)$ time to computes an n-dimensional signal. For now, any algorithm for computing the exact DFT is $\Theta(n)$ time, which need to take time at least proportional to its output size. In many information applications, signal has a certain degree of sparsity or compressibility. And it exists in many areas such as image processing[1], compressed sensing[2, 3], computational learning theory[4, 5], multi-scale analysis[7], etc.. Because of the specific structure and existed widely, this kind of signal has become a hot research field in the area of signal processing. For Fourier transform, several sublinear algorithms were presented [4, 7-10], but most of them are only applicable to very sparse signals because of the structure limitation.

Peer-review under responsibility of the scientific committee of the 7th International Congress of Information and Communication Technology
doi:10.1016/j.procs.2017.03.176

Sparse Fast Fourier Transform (SFFT) algorithm is a hot research direction of sparse signal. The algorithmic runtime is sublinear in the signal size. This makes it possible to have a higher speed in signal processing. Some algorithms was proposed in recent years. The research of sparse fast Fourier Transform focuses on the minimizing the time complexity [11], or extends to higher dimensions [12, 13]. Pervious researches demand higher support for signal. And there is no deep research in the parameters and applicability of signal.

The paper discusses the desired conditions of reconstruct original signal Fourier spectrum under the signal environment of additive white Gaussian noise. The structure of the paper is as follows: The first part of this paper is the introduction. Then the second part introduces the sparse fast Fourier transform algorithm; statistical characteristics of the signal after the process of reducing dimensions is analyzed and deduced in the following part; the fourth part is the simulation, and the conclusion would be drawn in the final part.

2. Sparse Fast Fourier Transform Theory

The sparse fast Fourier transform theory adopts some methods of dimension reduction to process frequency-sparse signals in time domain, which compress frequency domain information from high-dimension to low-dimension. Then do the fast Fourier transform. In the end, reconstructing the Fourier spectrum of original signal by location and estimation. The block diagram of SFFT is shown below:

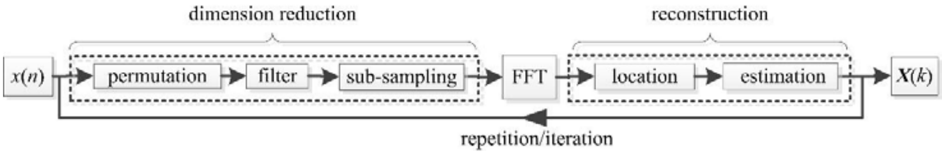

Fig. 1. The block diagram of sparse fast Fourier transform algorithm.

2.1. sparse representation of signals

If a signal can be processed by sparse fast Fourier transform, this signal could be sparse representation in frequency domain. So it could reduce to low dimensional space to reduce the computation complexity. For a discrete input signal $x \in \mathbb{R}^{N\times 1}$, it could be represented with linear combination of orthogonal basis vectors $\{\boldsymbol{f}_i\}_{i=1}^{N}$. Let $\boldsymbol{F} = [\boldsymbol{f}_1 \boldsymbol{f}_2 \cdots \boldsymbol{f}_N]$ be a set of orthogonal basis vectors in frequency domain $\mathbb{R}^{N\times 1}$, then x could be represented as

$$\boldsymbol{x} = \boldsymbol{Fs} \tag{1}$$

where s is the coefficient vector of the orthogonal basis $\boldsymbol{F}$. If only *K* coefficients is not zero, the other *N-K* coefficients is zero, it would call this *K*-sparse in frequency domain. And its sparsity is *K*. Similarly, if *K* coefficients is very large, the other *N-K* coefficients could be negligible or equal to zero, then it would call this approximately *K*-sparse. In this paper, the signal contain noise, so the signal is approximately *K*-sparse.

2.2. dimension reduction in time domain

Since the signal is sparse in frequency domain, the frequency could divide into different buckets with a certain rules. Hence, each frequency of signal will be guaranteed to exist in a bucket. In frequency domain, each bucket of signal would be regarded as a new unit to process, so a long sequence could be mapped to short sequence. And it can runn with FFT in the short sequence, and the algorithm complexity would be greatly reduced. The process of reducing dimension in time domain of SFFT could be expressed in the following:

$$y = \boldsymbol{\Psi}_{B\times N} x \tag{2}$$

where $\boldsymbol{\Psi}_{B\times N}$ is the matrix representation in the process of reducing dimensions of signal $\boldsymbol{x}$ in time domain, and $\boldsymbol{y}$ is time vector of sub-sampling, B is bucket of signal. The process of reducing dimensions in time domain would mainly be divided into three parts: permutation of spectra, filter and sub-sampling.

In fact, the implementation of processing that regard per bucket as an isolate unit is choosing an effective