Reliability Engineering and System Safety ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Optimal redundancy allocation to maximize multi-state computer network reliability subject to correlated failures

Cheng-Ta Yeh [a,*], Lance Fiondella [b]

[a] Department of Business Administration, Shih Hsin University, Taipei, Taiwan
[b] Department of Electrical & Computer Engineering, University of Massachusetts, Dartmouth, MA, USA

ARTICLE INFO

Article history:
Received 17 April 2016
Received in revised form
21 August 2016
Accepted 28 August 2016

Keywords:
Physical line redundancy allocation
Multi-state computer network
Reliability maximization
Correlated binomial distribution
Simulated annealing

ABSTRACT

Modern society depends on the stability of computer networks. One way to achieve this goal is to determine the optimal redundancy allocation such that system reliability is maximized. Redundancy requires that each edge in computer networks possess several binary-state physical lines allocated in parallel. A computer network implementing redundancy allocation is called a multi-state computer network (MSCN), since each edge can exhibit multiple states with a probability distribution according to the number of binary-state physical lines that are operational. However, past research often fails to consider the possibility of correlated failures. This study applies a correlated binomial distribution to characterize the state distribution of each edge within a network and a redundancy optimization approach integrating simulated annealing (SA), minimal paths, and correlated binomial distribution is proposed. The approach is applied to four practical computer networks to demonstrate the computational efficiency of the proposed SA relative to several popular soft computing algorithms.

1. Introduction

In modern society, computer networks are the main medium for data transmission. Accidents or repair may disrupt network operation, preventing the execution of normal operations. Preserving network stability is thus important to ensure uninterrupted service. Therefore, many system supervisors focus on system reliability assessment and maximization for the computer networks they operate. The issues underlying system reliability assessment can be traced to binary-state computer network reliability assessment. In a binary-state computer network, each edge denoting a transmission line, may either work or not. System reliability is defined as the probability that the source vertex is connected to the sink vertex [6,16]. Reliability assessment can also be extended to multi-state computer networks (MSCNs), where each edge is composed of several physical lines and each physical line may provide a fixed capacity (i.e. bandwidth) or fail. Clearly, such MSCNs are more detailed than their binary state counterparts. In the multi-state systems, reliability is defined as the probability that a computer network can successfully transmit d units of data from source to sink [17,18,34].

Since system reliability is regarded as a performance index to evaluate the capability of a computer network, most studies discuss system reliability maximization for MSCNs from the perspective of network topology optimization [5]; [28] or component assignment optimization [19,21]. Network topology optimization focuses on exploring the connections between edges and vertices to search for the optimal network topology that maximizes system reliability. According to Stevenson [30], topology optimization is often expensive since it involves designing or redesigning networks. Component assignment optimization considers a set of multi-state components available for allocation to edges with the goal of determining the optimal component assignment that maximizes system reliability. The advantage of component assignment optimization is that system reliability can be improved without altering the existing network topology. However, in such situations, each component's maximal redundancy is fixed (i.e. each component's redundancy level is given) such that an edge may provide surplus capacity after allocating a component.

Redundancy allocation is another way to maximize system reliability for computer networks. The concept of redundancy allocation was introduced by Misra and Sharma [24]. It requires that each edge consists of several binary-state components allocated in parallel to enhance system stability. In other words, redundancy allocation determines the redundancy level for each edge and has been applied in many fields. Huang et al. [12] adopted a redundancy technique to promote the reliability for an aircraft's multi-channel electrical power supply system. Tian et al. [31] presented an optimization model for a multi-state series-parallel

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[*] Corresponding author.
E-mail addresses: ctyeh@mail.shu.edu.tw (C.-T. Yeh), lfiondella@umassd.edu (L. Fiondella).

http://dx.doi.org/10.1016/j.ress.2016.08.026
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Please cite this article as: Yeh C-T, Fiondella L. Optimal redundancy allocation to maximize multi-state computer network reliability subject to correlated failures. Reliability Engineering and System Safety (2016), http://dx.doi.org/10.1016/j.ress.2016.08.026

Nomenclature

$\mathbf{E}$; n	set of edges connecting pair of vertices; number of edges
$\mathbf{V}$	set of vertices
$(\mathbf{E}, \mathbf{V})$	computer network
O, D	unique source vertex; unique sink vertex
e_i	edge #i for $i=1, 2, \ldots, n$
k	capacity of a physical line
p	successful probability of physical line
c	cost per unit of length of physical line
y_i	redundancy level of edge e_i, $i=1, 2, \ldots, n$
$\boldsymbol{Y}$	$(y_1, y_2, \ldots, y_n)$: a redundancy allocation or solution
$C_{\boldsymbol{Y}}$	allocation cost associated with $\boldsymbol{Y}$
$G_{i,w}$	state #w of edge e_i, $w=1, 2, \ldots, y_i+1$, where $G_{i,1}=0$
$\hat{G}_i$	maximum capacity of edge e_i
x_i	current capacity of e_i, $i=1, 2, \ldots, n$
$\boldsymbol{X}$	$(x_1, x_2, \ldots, x_n)$: (current) capacity vector
$\boldsymbol{M}_{\boldsymbol{Y}}$	$(\hat{G}_1, \hat{G}_2, \ldots, \hat{G}_n)$: maximal capacity vector associated with $\boldsymbol{Y}$
$\mathbf{U}_{\boldsymbol{Y}}$	set of $\boldsymbol{X}$ feasible associated with $\boldsymbol{Y}$
d	demand at the sink vertex D
$\mathbf{X}_{\boldsymbol{Y}}$	set of $\boldsymbol{X}$ fulfilling d
$R_d(\boldsymbol{Y})$	system reliability given component allocation $\boldsymbol{Y}$
B	allocation budget
k	transmission capacity per unit of redundancy level
ρ_i	correlation among physical lines along edge e_i
γ	performance level of an edge
β_i	notational simplification
m	number of MPs
P_j	MP #j, $j=1, 2, \ldots, m$
f_j	amount of flow through P_j, $j=1, 2, \ldots, m$
$\mathbf{F}$	$(f_1, f_2, \ldots, f_m)$: data flow pattern
$\mathbf{F}_{\boldsymbol{MY}}$	set of F feasible under $\boldsymbol{M}_{\boldsymbol{Y}}$
$\mathbf{F}_{\boldsymbol{X}}$	set of F feasible under X with $X\in\mathbf{U}_{\boldsymbol{Y}}$
τ	iteration #τ
θ	number of neighbors
PR	probability to accept the best neighborhood solution
T_0	initial temperature
T_τ	control temperature in iteration #τ
α	cooling rate
Ψ	termination time

system to jointly determine the optimal component state distribution and optimal redundancy for each stage. Azaron et al. [3] solved a multi-objective discrete reliability optimization problem in a cold-standby redundant system using a genetic algorithm (GA). Tian et al. [32] developed a practical approach for the joint reliability–redundancy optimization of multi-state series-parallel systems. The approach not only determines the optimal redundancy level for each parallel subsystem, but also finds the optimal values for the variables that affect the component state distributions in each subsystem. Liu et al. [22] presented an approach of joint redundancy and imperfect maintenance strategy optimization for multi-state systems. Feizollahi et al. [8] proposed a robust optimization framework to deal with uncertain component reliabilities in redundancy allocation problems for series-parallel systems. Roy et al. [29] discussed a multi-objective reliability redundancy allocation problem for the series-parallel system and found the optimum number of redundant components to maximize system reliability and minimize system cost with entropy as the constraint. These studies demonstrated that an advantage of redundancy allocation is that each edge's redundancy level is appropriately determined so that the edge has no surplus capacity as much as possible.

In this study, redundancy allocation of an edge determines the number of physical lines allocated to the edge, i.e. redundancy level. For example, an edge with redundancy level 2 combines 2 physical lines. Since each physical line possesses two states, the edge has three states (i.e. no physical line works, one of physical line works, and both physical lines work) following a binomial distribution. Most studies [17-19,21] assume the physical lines allocated to an edge are statistically independent. However, failure of a physical line may cause other lines in the edge to partially or completely fail. For example, a disaster may cause multiple physical lines to fail, necessitating repair. During repair of a physical line, other lines belonging to the edge need to be partially or fully suspended. Such a phenomenon is called a correlated failure and should not be ignored in system reliability assessment and optimization [9,10].

This paper determines the optimal redundancy allocation of a computer network to maximize system reliability subject to a budget, while considering the impact of correlated physical-line failures. To our knowledge, this problem has not been considered previously. Chambari et al. [4] illustrated that redundancy allocation optimization is a NP-hard problem. Thus, the problem considered here is also NP-hard. In order to solve the optimal redundancy allocation problem with correlated failures, a simulated annealing (SA) approach is proposed based on the conceptual approach of Chambari et al. [4]. SA was introduced by Metropolis et al. [23] and has been widely applied to combinatorial optimization problems [33]. SA stimulates the annealing process of solids to find a near optimal solution and has the advantage of avoiding convergence to local optimum. According to the strategy suggested by Dong et al. [7], we integrate 1-opt and 2-opt operations in the stage of neighborhood search of the proposed SA to enhance its ability to perform global search. Moreover, a correlated binomial distribution model developed by Fiondella and Zeephongsekul [10] is utilized to evaluate the probability distribution for each edge under redundancy allocation when failures are correlated, and the system reliability is evaluated in terms of minimal paths (MP). The purposes of this study are to propose and solve the system reliability redundancy allocation problem with correlated failures and to demonstrate the computational efficiency of the proposed SA by comparing it with GA, particle swarm optimization (PSO), and tabu search (TS) through case studies of four practical computer networks.

The remainder of the paper is organized as follows. Assumptions and problem formulation are detailed in Section 2. State distribution of a correlated physical line redundancy allocation is illustrated in Section 3. Multi-state computer network model associated with a redundancy allocation is constructed in Section 4. Section 5 describes the proposed SA. Numerical experiments of four practical computer networks are executed to compare the proposed SA with several popular soft computing algorithms in Section 6. The conclusions are given in Section 7, along with future research.

2. Assumptions and problem formulation

Let $\mathbf{E}=\{e_i|1\le i\le \mathrm{le};n\}$ be a set of edges, where e_i denotes the ith edge and $\mathbf{V}$ be a set of vertices. Thus, a computer network is represented as $(\mathbf{E}, \mathbf{V})$. The length of e_i is denoted by l_i for $i=1, 2, \ldots, n$. Moreover, let k be the capacity of a single physical line. The

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