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Clustering algorithms for Risk-Adjusted Portfolio Construction

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Abstract

This paper presents the performance of seven portfolios created using clustering analysis techniques to sort out assets into categories and then applying classical optimization inside every cluster to select best assets inside each asset category.

The proposed clustering algorithms are tested constructing portfolios and measuring their performances over a two month dataset of 1-minute asset returns from a sample of 175 assets of the Russell 1000® index. A three-week sliding window is used for model calibration, leaving an out of sample period of five weeks for testing. Model calibration is done weekly. Three different rebalancing periods are tested: every 1, 2 and 4 hours. The results show that all clustering algorithms produce more stable portfolios with similar volatility. In this sense, the portfolios volatilities generated by the clustering algorithms are smaller when compare to the portfolio obtained using classical Mean-Variance Optimization (MVO) over all the dataset. Hierarchical clustering algorithms achieve the best financial performance obtaining an adequate trade-off between accumulated financial returns and the risk-adjusted measure, Omega Ratio, during the out of sample testing period.

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1 Introduction

In finance, portfolio construction or asset allocation is one of the most frequent problems practitioners solve every day. Portfolio Theory by Markovitz [10] introduced the problem faced by investors on a daily basis, in a framework called mean-variance as an optimization problem, specifically minimizing portfolio variance at a given level of expected or minimum required return. Markovitz summarized the solution space using the minimum variance frontier or more precisely, the positive slope section commonly known as the efficient frontier.

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Markovitz's framework is presented from two perspectives. Given a portfolio's expected variance σ_p, find the maximum return μ_p; on the other hand, given a portfolio's expected return μ_p, find the minimum variance σ_p. The two approaches are consistent. The optimization problem is built assuming a portfolio consisting of n risky stocks, and positive definite VCV matrix σ, where VCV is a Variance-Covariance matrix. The objective is to find a weight vector w that minimizes portfolio's total variance. This approach is known as the mean-variance optimization (MVO).

Hence, Markovitz's portfolio optimization find a global minimum for the following objective function:

$$f(w) = V(w, \sigma) = w'\sigma w \tag{1}$$

subject to the general constraint, portfolio weights must sum up to one.

This model is commonly associated with the Capital Asset Pricing Model (CAPM) developed by William Sharpe [14]. This fact partially explained why Harry Markowitz shared the Nobel Prize in 1990 with Sharpe. However, the two models are used for different purposes by financial practitioners.

CAPM theory considers Markowitz's model from a microeconomics perspective to discover price formation of financial assets. In this model, the central concept is that market portfolio is uniquely defined. In Markowitz's model, portfolio optimization depends on expected or preferred returns and risks. Furthermore, the optimal portfolio is not unique and depends on investor's risk aversion. As a consequence, these two models could give two different approaches to the asset allocation problem. While the CAPM theory is the principal pillar for passive management, Markowitz's model is the central technique to start actively managing a portfolio if a practitioner believes that the information set is not unique or homogeneously spread across market participants.

Analytically, let us consider a universe of n risky assets. Let $\mu = \mathbb{E}[R]$ and $\sum = \mathbb{E}[(R-\mu)(R-\mu)^T]$ be the vector of expected returns and the asset's return covariance matrix. Classical MVO assumes that a portfolio with weights $w = (w_1, w_2, ..., w_n)^T$ of n risky assets is fully invested, meaning that $\sum_{i=1}^{n} w_i = 1$, hence, short sells are not allowed, i.e. $w > 0$. When we have n stock prices time series: S_{ij} at time j for security i, the log returns are $R_{ij} = \ln \frac{S_{ij}}{S_{ij-1}}$, then is possible to denote $R = (R_1, \cdots, R_n)$ the asset returns vector. Portfolio's return is equal to $R(w) = \sum_{i=1}^{n} w_i R_i$. In a matrix form, it is obtained $R(w) = w^T R$, portfolio's expected return is:

$$\mu(w) = \mathbb{E}[R(w)] = \mathbb{E}[w^T R] = w^T \mathbb{E}[R] = w^T \mu \tag{2}$$

While its variance is equal to:

$$\begin{aligned}
\sigma^2(w) =& \mathbb{E}[(R(w) - \mu(w))(R(w) - \mu(w))^T] \\
=& \mathbb{E}[w^T R - w^T \mu)(w^T R - w^T \mu)^T] \\
=& \mathbb{E}[w^T (R - \mu)(R - \mu)^T w] \\
=& w^T \mathbb{E}[(R - \mu)(R - \mu)^T] w \\
=& w^T \sum w
\end{aligned} \tag{3}$$

On Figure 1, we have simulated 500 portfolios, with n=4 securities, and 1,000 observations for each asset. It is possible to see the minimum variance frontier, shown by the dotted green line, the positively sloped segment contains all the optimum portfolios for a given level of desired risk.