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Improving electromagnetism algorithm for solving resource allocation problem in stochastic networks

Weng-Ming Chu [a], Koan-Yuh Chang [b,*]

[a] Department of Electrical Engineering, National Chin-Yi University of Technology, Taiwan
[b] Department of Electronic Engineering, Chienkuo Technology University, Taiwan

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ABSTRACT

This study investigates optimal resource allocation for minimizing total cost in stochastic networks (SN) where the duration of all activities involved is not only a random variable, but also a function of the resources allocated. The total cost of the network comprises resource usage cost and penalty cost. An Electromagnetism Algorithm (EA) is used as a decision tool for optimization and a Label-Correcting Tracing Algorithm (LCTA) for approximation of completion time in SN is suggested. Furthermore, the Critical Path Cluster Algorithm (CPCA) and Cluster Local Search Algorithm (CLSA) are developed to enhance EA's search ability for resource allocation. Results from numerical experiments show that the proposed EA yields good solution quality.

1. Introduction

This study investigates optimal resource allocation in SN. In actual practice, project managers usually make decisions on allocating limited available resources to activities in order to complete the project with the minimum expected total cost. The project will be subject to the given precedence constraints among the activities and the project due day. Such problem is named herein as the Optimal Resource Allocation Problem of Stochastic Networks (ORAPSN). Note that the duration of activities involved is not only a random variable, but also a function of the resources allocated. Hence, ORAPSN is a difficult optimization problem for decision-makers because of the probabilistic characteristics of SN, e.g., the project completion time. There is a great deal of uncertainty regarding the work content of the activities which is expressed in the type of a probability distribution function. The randomness of the duration of activities will directly impact the total cost and completion time of the project. In face of such uncertainty of the work content, the project manager still has to decide upon a method of resource allocation for each of the activities. The duration of an activity then becomes the consequence of the resources allocated to it. One may allocate more resources to shorten the duration of an activity with the objective of avoiding penalty cost (see (Dawson and Dawson, 1998; Tavares and Ferreira, 1998; Ward and Chapman, 2003)). On the other hand, resource allocation will also introduce extra costs to the network. Therefore, there exists a trade-off relation between resource allocation and duration of activity. The purpose of ORAPSN is to determine the minimum expected total cost, and to attain the optimal performance of the project network that maintain both resources and duration of activities in proper realization (Basso and Peccati, 2001).

The above problem is closely related to the Time-Cost Tradeoff Problem (TCTP). Although De et al., (1995) had conducted a thorough survey on TCTP, most of the literature is about deterministic networks. Feng et al. (2000) classified the solution methods into mathematical programming and heuristic approaches. Mathematical programming utilizes the analysis of linear programming, integer programming, dynamic programming or the hybrid approach (see (Burns et al., 1996; Henderickson and Au, 1989; Patterson and Huber, 1974; Robinson and Huber, 1975)). Studies such as those conducted by Demeulemeester et al. (1993), Falk and Horowitz (1972) and Fulkerson (1961) used various kinds of cost functions for analysis. On the other hand, the heuristic approaches include series heuristic and parallel heuristic methods (see (Bell and Han, 1991; Boctor, 1993; Kahattab and Choobinch, 1991)).

This study also focuses on time-cost tradeoff optimization in SN. This problem has been proved to be difficult and there were only few studies on this issue, most of which involved dynamic programming (see (Basso and Peccati, 2001; Elmaghraby, 1993; Hinddelang and Muth, 1979; Robinson and Huber, 1975; Tereso et al., 2004a)). Other methods, as used by Weglarz (1981) and Azaron et al. (2006), employed the optimal control heory and assumed all activity durations as exponential or erlang distributions. The above works are all classified as analytical approaches which usually de-

* Corresponding author.
E-mail address: kychang@cc.ctu.edu.tw (K.-Y. Chang).

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mand complex and enormous computational efforts for ORAPSN or even medium-sized activity networks. Ghomi and Ashjari (2002) developed a simulated model for multi-project resource allocation. The simulation-based approach encountered the same problems as analytical approaches, because simulation requires tremendous running time for approximating the objective function. Therefore, the most feasible solution methodology for ORAPSN would be the heuristic approach. The population-based heuristic approach, developed from principles of evolution and heredity, has garnered growing interest in optimal problem-solving. Such evolutionary algorithms for computation maintain a population of potential solutions, iterate a specified *selection process* and *mutation operator* to improve the quality of solutions, and generate the near-optimal solution. Genetic Algorithm (GA) is one of the most popular methods, suggested by Azaron et al. (2006), for solving ORAPSN. It employs the analytical method of Kulkarni and Adlakha (1986) to approximate the project completion time but the activity durations are restricted to exponential or erlang distributions. Tereso et al. (2004b) also solved ORAPSN by applying the EA developed by Birbil and Fang (2003). However, Monte Carlo Simulation (MCS) is adopted for approximating the fitness values, which would be very inefficient and impractical due to the heavy burden on computation time.

Furthermore, this study proposes an improved EA as the decision tool for solving this stochastic optimization problem. In the proposed EA, the fitness values of the population samples are approximated by the LCTA (see (Yao et al., 2007)) that can eliminate computational burdens. Beyond that, the EA still confronts the immense domain range of solution-searching for larger networks, which would prolong the running time of finding the optimal solution and, finally undermine the efficiency of EA. Therefore, two heuristic algorithms are developed from the concept of "Critical Path Cluster (CPC)", which is a subset of paths in SN and dominates the determination of project completion time. The first heuristic algorithm called CPCA can efficiently generate the initial solutions which are close to the domain area where the optimal solution is possibly located. The second heuristic algorithm called CLSA is a local search procedure that assists the proposed EA to effectively obtain better solutions in the neighborhood of the sample points.

The rest of this paper is organized as follows. Section 2 defines the problem and discusses the formulation model of the ORAPSN. Section 3 describes the theory and the procedures involved in the EA. Section 4 introduces the concept of CPC; and the proposed heuristic algorithms, CPCA and CLSA are presented in Sections 5 and 6 respectively. In Section 7, 14 different instance networks used by Tereso et al. (2004b) are reused by our proposed method and the findings compared with previous ones. Finally, some conclusions are given in Section 8.

2. Problem definition

As stated above, the duration of an activity would be the consequence of the resources allocated; it is not only a random variable but also a specified function of the resources allocated. Therefore, the resource allocation pattern of the network is closely related to the total cost of project, which comprises usage cost of resources allocated and penalty cost. From this point of view, the resource allocation issue and the costs it incurred in the SN become the foundation for solving ORAPSN.

2.1. Resource allocation in SN

Given a project composed of N activities, the duration of an activity i is specified as a random variable and denoted by $Y_i(t), i=1, ..., N$. Let R_i denote the allocated resource which is defined

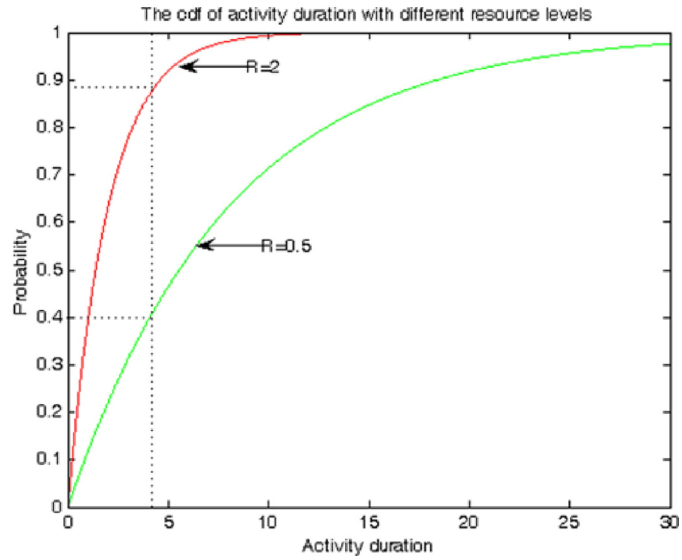

Fig. 1. Probability curve of activity duration with different resource levels.

in Eq. (1).

$$l_i < R_i < u_i, i = 1, ..., N \tag{1}$$

Where l_i and u_i are the lower bound and upper bound of R_i, respectively. Obviously, $Y_i(t)$ would be the function of the resource allocated R_i. Assume that all the activities are exponentially distributed for convenience of explanation. In this study, there is no restriction on the type of variable distribution); therefore, $Y_i(t)$ can be represented simply as:

$$Y_i(t) = \lambda_i \cdot R_i e^{-\lambda_i R_i t}, mean(Y_i(t)) = \frac{1}{\lambda_i R_i} \tag{2}$$

where λ_i is the parameter of the exponential distribution function of activity i. With the above equation, one can easily note that if the allocated resource level exceeds 1, the expected duration decreases; otherwise, the expected duration increases. For example, let $Y_i(t) = 0.25R_i e^{-0.25R_i t}$ and refer to Fig. 1. If the project manager allocates resource $R_i=0.5$ to activity i, i.e., $R_i=0.5$, the probability for activity i to be completed in 4.5 time-units would be about 40%. For $R_i=2$, the probability increases up to 89%.

2.2. Performance of costs

Performance is defined in terms of total cost of project which comprises the following two terms. a. Considering the resource usage cost C_{R_i} of activity i with the allocated resourceR_i, its marginal value is usually normalized at U_C per unit time of activity duration. To evaluate this cost, one has to express the cost as a function of the intensity at which the resource is utilized. In general, it is of the form:

$$C_{R_i} = R_i^{\beta} \cdot Y_i(t) \cdot U_C, i = 1, ..., N \tag{3}$$

where β is determined empirically according to past experience. For simplicity of exposition, let $\beta=2$, so that the resource usage cost is quadratic in the intensity of resource application. Typically, $\beta<2$ implies that doubling the resource intensity incurs lower cost, which is four times less than the original cost. Substituting $\beta=2$ and summing over all resources yield the cost of resource usage expressed as:

$$C_U = \sum_{i=1}^{N} C_{R_i} = \sum_{i=1}^{N} R_i^2 \cdot Y_i(t) \cdot U_C \tag{4}$$