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Price of anarchy and an approximation algorithm for the binary-preference capacitated selfish replication game ☆

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ABSTRACT

We consider the capacitated selfish replication (CSR) game with binary preferences, over general undirected networks. We study the price of anarchy of such games, and show that it is bounded above by 3. We develop a quasi-polynomial algorithm $\mathcal{O}(n^{2+\ln D})$, where n is the number of players and D is the diameter of the network, which can find, in a distributed manner, an allocation profile that is within a constant factor of the optimal allocation, and hence of any pure-strategy Nash equilibrium (NE) of the game. Proof of this result uses a novel potential function. We further show that when the underlying network has a tree structure, every globally optimal allocation is an NE, which can be reached in only linear time. We formulate the optimal solutions and NE points of the CSR game using integer linear programs. Finally, we introduce the LCSR game as a localized version of the CSR game, wherein the actions of the players are restricted to only their close neighborhoods.

1. Introduction

Due to accelerated growth of economic networks and advances in game theory and distributed control driven by various applications, modeling of distributed network storage has become an important research area in recent years. In fact, resource allocation is one of the primary topics frequently addressed in the literature, emerging in various disciplinary areas such as economics, computer science, epidemiology, and engineering.

In general, distributed network storage games or resource allocation games are characterized by a set of agents who compete for the same set of resources (Masucci & Silva, 2014; Pacifici & Dan, 2012), and arise in a wide variety of contexts such as congestion games (Ackermann, Röglin, & Vöcking, 2008; Anantharam, 2004; Fabrikant, Papadimitriou, & Talwar, 2004; Milchtaich, 1996), load balancing (Caragiannis, Flammini, Kaklamanis, Kanellopoulos, & Moscardelli, 2006; Ghosh & Muthukrishnan, 1994), peer-to-peer systems (Friedman, Halpern, & Kash, 2006; Laoutaris, Telelis, Zissimopoulos, & Stavrakakis, 2006; Pollatos, Telelis, & Zissimopoulos, 2008), web-caches (Gopalakrishnan et al., 2012), content management (Pollatos et al., 2008), auctioning and mechanism design (Maheswaran & Başar, 2001, 2003), and market sharing games (Goemans, Li, Mirrokni, & Thottan, 2006). Among many problems that arise in such a context, one that stands out is distributed replication, which not only improves the availability of resources for users, but also increases the reliability of the entire network with respect to customer requests (Chun et al., 2004; Goyal & Vega-Redondo, 2000). However, one of the main challenges in modeling resource allocation problems using game-theoretic tools is to answer the question of to what extent such models can predict the desired optimal allocations over a given network. Modeling a system as a game, ideally one would like the set of Nash equilibria of the game to be as close as possible to some target states that one is seeking in the system (Johari, Mannor, & Tsitsiklis, 2005; Marden & Roughgarden, 2010). In fact, the *price of anarchy (PoA)* (Koutsoupias & Papadimitriou, 1999) is one of the metrics in game theory that measures efficiency and the extent to which a system degrades due to selfish behavior of its agents; it has been widely used in the literature (Chun et al., 2004; Goemans et al., 2006; Pollatos et al., 2008; Vetta, 2002).

Distributed replication games with servers that have access to all the resources and are accessible at some cost by users have been studied in Laoutaris et al. (2006). Moreover, the uncapacitated selfish replication game where the agents can hold as many resources as they wish in their caches by paying some extra fees

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was studied in Chun et al. (2004), where the authors were able to characterize the set of NE points in terms of the parameters of the problem. However, unlike the uncapacitated case where there is no restriction on the cache sizes, there is no comparable characterization of NE points in capacitated selfish replication (CSR) games where the agents are restricted to hold at most certain a fixed number of resources. In fact, when the agents have limited capacity, the situation could be much more complicated as the constraint couples the actions of agents much more than in the uncapacitated case or in replication games with servers.

Typically, CSR games are defined in terms of a set of available resources for each player, where the players are allowed to communicate through an undirected communication graph. Such a communication graph identifies the access cost among the players, and the goal for each player is to satisfy his/her customers' needs with minimum cost. Ideally, and in order to avoid any additional cost, each player only wants to use his/her own set of resources. However, due to capacity limitations, the players cannot have immediate access to all the resources they may need, and hence, they will have to borrow some of the resources which are not available in their own caches from others in order to meet their customers' demands, and such a transaction incurs cost. In fact, the motivation behind introducing the CSR game is driven by three facts which may arise in most of the resource allocation problems: myopic behavior of the agents aligned with their individual benefit, higher access cost to the resources which are farther away, and budget constraint on the number of resources that each agent can replicate (Correa, Schulz, & Stier-Moses, 2004; Nisan, Roughgarden, Tardos, & Vazirani, 2007). The problem of finding an NE for CSR games in the case of hierarchical networks was studied in Gopalakrishnan et al. (2012). Also, the class of CSR games with binary preferences has been studied in Etesami and Başar (2014), where "binary preferences" capture the behavioral pattern where each player has a set of objects he or she is equally interested in and has no interest in any of the remaining ones.

This paper is a major effort in the direction of substantially extending the set of results available on CSR games. We concentrate on games with binary preferences. In the model we adopt, players act myopically and selfishly, while facing the requirement to fully satisfy their customers' needs. Since there is no finer preference ordering among the different resources that are preferred, the goal for each player is to cache some resources which leads to minimization of her own cost. Note that, although each player acts in a selfish manner with respect to others, their actions are fairly tightly coupled, and thus they do not have absolute freedom in the selection of their individual actions.

Unlike the earlier results on this problem (Etesami & Başar, 2014; Gopalakrishnan et al., 2012), which in general provide only exponential time search algorithms for finding a pure-strategy NE (unless the number of resources is 2 Gopalakrishnan et al., 2012, or the underlying network satisfies some edge density assumption, in which case a polynomial time algorithm exists Etesami & Başar, 2014), in this work we consider CSR games over general undirected networks and devise a quasi-polynomial algorithm which drives the system to an allocation profile whose total cost lies within a constant factor of that in any pure-strategy NE. In particular, we provide the first upper bound on the price of anarchy (PoA) of such games, i.e., the ratio of the highest cost among all NE points to the overall minimum cost of an optimal allocation, and show that PoA is in general bounded above by 3. We note that, this work constitutes a substantial extension of the results in the conference paper (Etesami & Başar, 2015), as we show that when the underlying network is a tree, every optimal allocation which minimizes the overall sum cost must be an NE and provide a linear time algorithm to deliver such an optimal allocation. Further, we formulate the set of optimal allocations and NE points in closed form using an integer linear program (ILP) and extend the CSR game to a local CSR (LCSR) game which is a local version of the former, which is more relevant to real applications.

It is worth noting that the CSR game we consider in this paper is closely related to the graph coloring game (Panagopoulou & Spirakis, 2008), in which the resources can be interpreted as different colors and the question of interest is to obtain an NE (proper coloring) efficiently. However, the form of utilities and the interaction among the players make these two games totally different in nature. For instance, in the graph coloring game introduced in Panagopoulou and Spirakis (2008), the utilities of the players only depend on their immediate neighbors and the number of different colors. This allows the coloring game to admit an exact potential function,[1] namely the "variance" of the colors, which in turn allows the authors to find an NE in polynomial time. However, in our problem not only the number of resources and distribution of them are important, but also their exact positioning in the network is critical. This couples the actions and utilities of the players much more than that in the graph coloring game. As a result there is no comparable exact potential function with polynomial decrements at hand for the CSR game. Moreover, the strong coupling in the CSR game makes the search state space for NE exponentially larger due to the depth of distances in the network. Nevertheless, unlike the negative approximability results given in Panagopoulou and Spirakis (2008) for finding an optimal NE (a proper coloring with minimum number of colors), in this paper we provide quasi-polynomial approximation results for the NE points of the CSR game.

The paper is organized as follows. In Section 2, we introduce in precise terms CSR games with binary preferences over general undirected networks. We review some salient properties of such games and revisit some relevant existing results. In Section 3, we provide an upper bound on the price of anarchy. In Section 4 we devise a quasi-polynomial algorithm which can reach an allocation profile within a constant factor of the optimal one, and hence of any pure-strategy NE. In Section 5 we show that globally optimal solutions of the CSR game constitute a subclass of the set of NE points when the underlying network is a tree. In Section 6, we associate an integer linear program with the problem of obtaining the optimal allocation and NE points of the CSR game. We conclude the paper with the concluding remarks of Section 7. Finally, in the Appendix, we provide a local formulation of the CSR game when the actions of the players are restricted to their small neighborhoods.

Notations: We let $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{R}$ denote, respectively, the sets of positive integers, integers, and real numbers. For $n \in \mathbb{N}$, we let $[n] := \{1, 2, \dots, n\}$. For $\mathbf{v} \in \mathbb{R}^n$, we let $\mathbf{v}_i$ be the ith entry of $\mathbf{v}$, and $\mathbf{v}'$ be its transpose. We let $\mathbf{1}$ be the vector of all ones. We use $\mathcal{G} = ([n], \mathcal{E})$ to identify an undirected underlying network with a node set $[n]$ and an edge set $\mathcal{E}$. For any two nodes $i, j \in [n]$, we let $d_{\mathcal{G}}(i, j)$ be the graphical distance between them, that is, the length of the shortest path which connects i and j. The diameter of a graph, denoted by D, is the maximum distance between any pair of nodes, that is, $D = \max_{i,j\in[n]} d_{\mathcal{G}}(i, j)$. We let $\mathcal{N}(i)$ denote the set of neighbors of node i in the graph $\mathcal{G}$. Moreover, for an arbitrary node $i \in [n]$ and $r \in \mathbb{R}^{\geq 0}$, we define a ball of radius r and center i to be the set of all the nodes in the graph $\mathcal{G}$ whose graphical distance to node i is at most r, i.e., $B_{\mathcal{G}}(i, r) = \{x \in [n] | d_{\mathcal{G}}(i, x) \leq r\}$. For a vector $(s_1, s_2, \dots, s_n)$ we occasionally write (s_i, s_{-i}), where s_{-i} is the set of all entries except the ith one. We denote a specific NE and an optimal allocation for a system by P^* and P^o, respectively. Finally, we use $|S|$ to denote the cardinality of a finite set S, and $\mathbb{I}_S$ to be its characteristic function.

[1] A potential function is exact if any change in the individuals' utilities due to a change of action exactly is equal to the change of potential function.