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A Branch-and-Price algorithm for a compressor scheduling problem

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ABSTRACT

This work presents a Branch-and-Price algorithm for solving a compressor scheduling problem with applications in oil production. The problem consists in defining a set of compressors to be installed for supplying the gas-lift demand of oil wells while minimizing the associated costs. Owing to the non-convex nature of the objective function, two piecewise-linear formulations are tested in the pricing subproblem, which is solved with a two-phase strategy. Also, two branching strategies are proposed based on the original problem variables, and a specific rule is created for solving the master problem as an integer program for obtaining feasible solutions. Experimental results are reported for three sets of instances, for which the branch-and-price algorithm obtained more optimal solutions, and spent less time on average than the CPLEX solver applied to the piecewise-linear formulation. Furthermore, for the solution of the largest instances within a limited computational time, the proposed branch-and-price algorithm found good feasible solutions, outperforming CPLEX.

1. Introduction

As the production of hydrocarbons from an oil field progresses in time, the reservoir internal pressure decreases to a point that is not sufficiently high to lift hydrocarbons to the surface or obtain profitable production levels. In this case, some artificial lifting method as the continuous gas-lift, or simply gas-lift, is needed. Fig. 1 illustrates the production process in a well operated with gas-lift. In this technique, natural gas is compressed by compressors and injected at the bottom of the production tubing at a certain rate and pressure, flowing into this tube through valves. The gas blends with the hydrocarbons, making it less dense. Consequently, the weight of the fluid column is reduced and the blend can flow to the surface.

Gas-lift is a widely used technique due to its robustness, wide range of operating conditions, and relatively low installation and maintenance costs (Hamedi & Khamehchi, 2012). An oil recovery plan is needed to coordinate the gas-lift operation of an oil field. With the aim of maximizing cumulative production, a recovery plan establishes the gas rate and injection pressure for the production wells over the lifespan of the reservoir. However, these injection rates can be overestimated to hedge against unpredicted events such as compressor failures or dynamic changes in the reservoir conditions. If a compressor produces more gas than the total demand from the wells it supplies, the excess must be exported (giving it another destination) or burned in the gas flare. Moreover, energy losses are incurred when the output pressure from a compressor is reduced before injection, invariably increasing production costs. These issues give rise to the problem of allocating a set of compressors to meet the demands of a set of wells.

The Compressor Scheduling Problem (CSP) was first addressed by Camponogara, Castro, and Plucenio (2007) and formulated as an extension of the Single-source Capacitated Facility Location Problem (SSCFLP) with two capacities. That work is characterized by a non-convex objective function and nonlinear constraints, for which a piecewise-linear formulation based on convex combination was proposed to solve the problem with an off-the-shelf mixed-integer linear programming solver. Also, a preliminary polyhedral study was performed for obtaining a compact formulation with valid inequalities based on knapsack cover inequalities. Camponogara, de Castro, Plucenio, and Pagano (2011) extends the work of Camponogara et al. (2007) and compare the nonlinear and the piecewise-linear formulations, showing that the latter was more effective in reducing the operating costs. Also, exact and approximate lifting procedures and exact and heuristic separation procedures were proposed. The results demonstrated that the use of valid inequalities can reduce computational time and memory significantly.

A revised formulation for CSP is proposed in Camponogara, Nazari, and Meneses (2012). The nonlinear constraints were replaced by a family of linear inequalities. The new formulation is tighter than the previous one, and the computational results demonstrate very expressive gains concerning computational time. Also, the polyhedral dimensionality was established for some conditions that depend on problem parameters, and cover valid inequalities were proposed.

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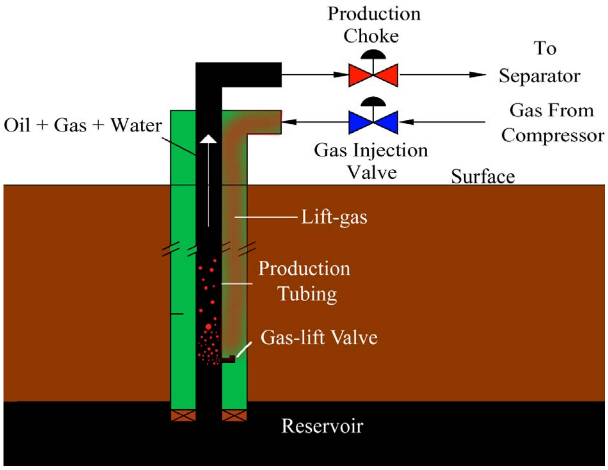

Fig. 1. Illustration of a well operated by the continuous gas-lift technique.

However, the gains with valid inequalities are less significant with the new formulation.

In Camponogara and Plucenio (2008) a column generation algorithm based on the Dantzig-Wolfe decomposition of a piecewise-linear formulation is proposed to yield a lower bound for the CSP problem. Friske, Buriol, and Camponogara (2015) adapted the column generation algorithm of Camponogara and Plucenio (2008) for the revised model of Camponogara et al. (2012). Computational experiments are reported to evaluate the lower bound quality provided by the column generation algorithm. With some modifications, the column generation algorithm was also tested under the single-source capacitated facility location problem.

In this work, we propose a Branch-and-Price algorithm (B&P) for solving the CSP with a piecewise-linear objective function. Experimental results are provided for three sets of instances, and CPLEX is applied to the same instances for comparison purposes. The experimental results demonstrate that the proposed B&P outperforms CPLEX in most of the instances, with respect to computational time and solution quality.

The remainder of this work is organized as follows. Section 2 presents the problem definition and its mathematical formulation. Section 3 presents the column generation algorithm for the CSP, including the piecewise-linear formulations for the pricing subproblem. Section 4 presents the proposed branch-and-price algorithm. Section 5 shows computational experiments. Finally, Section 6 presents the conclusions and future works.

2. Problem definition

In the Compressor Scheduling Problem, each well $i \in M$ has a demand of gas-lift rate q_i^{w} and pressure p_i^{w} for achieving the production levels defined by the recovery plan. The wells demand is supplied by gas-lift compressors, which are connected to the wells through pipelines. Each compressor $j \in N$ has a different operational range, such that the gas-lift compressing rate q_j^{c} can vary between $q_j^{\mathrm{c,min}}$ and $q_j^{\mathrm{c,max}}$. The pressure of gas-lift $p_j^{\mathrm{c}}(q_j^{\mathrm{c}})$ is a nonlinear function of the gas rate produced by compressor j and should be greater or equal to p_i^{w} of each supplied well plus the loss of pressure l_{ij} along the pipelines. Each compressor can supply one or more wells, and a well can be supplied by just one compressor. The activation of a compressor j incurs in a fixed cost c_j while supplying a well i has a maintenance cost e_{ij}. An operating cost $h_j^{\mathrm{c}}(q_j^{\mathrm{c}})$ is accounted for each activated compressor j, which is a product of the per unit of energy cost d_j,q_j^{c}, and $p_j^{\mathrm{c}}(q_j^{\mathrm{c}})$. The Compressor Scheduling Problem consists in defining, among a set of gas-lift compressors, which ones will be installed (or just activated) and how the compressors will supply gas-lift to the wells at their required rates and pressures, while minimizing the mentioned costs.

Table 1
Notation used in the CSP formulations.

Symbol	Definition
Sets	
$j \in N$	Set of compressors
$i \in M$	Set of wells
$j \in N_i$	Subset of compressors that can supply well i
$i \in M_j$	Subset of wells that can be supplied by compressor j
Parameters	
$n = \|N\|$	Number of compressors
$m = \|M\|$	Number of wells
c_j	Installation cost of compressor j
d_j	Per unity energy cost of compressor j
$q_j^{\mathrm{c,min}}$	Minimum output gas rate of compressor j
$q_j^{\mathrm{c,max}}$	Maximum output gas rate of compressor j
$\alpha_{l,j}, l \in \{0,\ldots,4\}$	Parameters of discharge pressure of compressor j
q_i^{w}	Gas rate demand of well i
p_i^{w}	Gas pressure demand of well i
e_{ij}	Cost of supply/maintenance between compressor j and well i
l_{ij}	Pressure loss in the pipeline between compressor j and well i
$q_j^{\mathrm{c,max},i}$	Maximum output gas rate q_j^{c} of compressor j at which the discharge pressure is sufficiently high to supply well i, i.e. $\max\{q_j^{\mathrm{c}}: q_j^{\mathrm{c,min}} \leqslant q_j^{\mathrm{c}} \leqslant q_j^{\mathrm{c,max}}, p_j^{\mathrm{c}}(q_j^{\mathrm{c}}) + l_{ij} \geqslant p_i^{\mathrm{w}}\}$
Variables	
$y_j \in \{0,1\}$	Indicates whether the compressor j is installed (1) or not (0)
$x_{ij} \in \{0,1\}$	Indicates whether the well i is supplied by compressor j (1) or not (0)
$q_j^{\mathrm{c}} \in \mathbb{R}_+$	Gas rate output of compressor j
Functions	
$p_j^{\mathrm{c}}(q_j^{\mathrm{c}})$	Discharge pressure output of compressor j, $p_j^{\mathrm{c}}(q_j^{\mathrm{c}}) = \alpha_{0,j} + \alpha_{1,j} q_j^{\mathrm{c}} + \alpha_{2,j}(q_j^{\mathrm{c}})^2 + \alpha_{3,j}(q_j^{\mathrm{c}})^3 + \alpha_{4,j}\ln(1 + q_j^{\mathrm{c}})$
$h_j^{\mathrm{c}}(q_j^{\mathrm{c}})$	Operating cost function of compressor j, $h_j^{\mathrm{c}}(q_j^{\mathrm{c}}) = d_j \cdot q_j^{\mathrm{c}} \cdot p_j^{\mathrm{c}}(q_j^{\mathrm{c}})$

For convenience, Table 1 gives the notation used in the problem formulation. The CSP can be formulated as a Mixed-integer Nonlinear Problem (MINLP) as follows:

$$\min \quad P = \sum_{j \in N} c_j y_j + \sum_{i \in M} \sum_{j \in N_i} e_{ij} x_{ij} + \sum_{j \in N} h_j^{\mathrm{c}}(q_j^{\mathrm{c}}) \tag{1a}$$

$$\text{s. t.:} \quad x_{ij} \leqslant y_j, \quad i \in M, j \in N_i \tag{1b}$$

$$\sum_{j \in N_i} x_{ij} = 1, \quad i \in M \tag{1c}$$

$$q_j^{\mathrm{c}} \leqslant q_j^{\mathrm{c,max},i} x_{ij} + q_j^{\mathrm{c,max}}(y_j - x_{ij}), \quad j \in N, i \in M_j \tag{1d}$$

$$q_j^{\mathrm{c}} \geqslant q_j^{\mathrm{c,min}} y_j, \quad j \in N \tag{1e}$$

$$\sum_{i \in M_j} q_i^{\mathrm{w}} x_{ij} = q_j^{\mathrm{c}}, \quad j \in N \tag{1f}$$

$$y_j \in \{0,1\}, \quad j \in N \tag{1g}$$

$$x_{ij} \in \{0,1\}, \quad i \in M, j \in N_i. \tag{1h}$$

The objective function (1a) minimizes the compressor installation and operation costs, and the supply/maintenance costs between compressors and wells. Constraint set (1b) determines that a well must be supplied by an installed compressor. Constraint set (1c) imposes that all wells must be supplied, each one by a single compressor. Constraint set (1d) imposes the upper limit of the gas rate q_j^{c}. Note that if $x_{ij} = 1$, just the first term in the right-hand side of the inequality is activated, i.e. the output gas rate q_j^{c} must be less than or equal to $q_j^{\mathrm{c,max},i}$. Otherwise, only the second term of the right-hand side is activated, meaning that compressor j must not exceed its maximum output gas rate $q_j^{\mathrm{c,max}}$.