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A reflection on analytical tornado-like vortex flow field models

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ABSTRACT

Given the difficulties associated with undertaking full-scale measurements in tornadoes, recourse is often made to models. In this field, analytical models have, perhaps surprisingly, stood the test of time, with the Rankine, Burgers-Rott and Sullivan models frequently invoked to model the flow field of a tornado. These mathematical models are by their very nature, a simplification of what is a highly complex phenomenon. However, in many cases they have been represented as the 'truth' without the fundamental assumptions governing the model being either explored in detail or even acknowledged. This paper attempts to rectify this by giving detailed information about assumptions and limitations of each vortex model and critically assesses the ability (or otherwise) of the Rankine, Burgers-Rott, Sullivan, and the recently published Baker vortex model to simulate tornado-like flow. Comparisons are made to the flow field of a physically simulated tornado, which by its very nature is also a model, but arguably more realistic.

It was found that the vortex models are able to represent certain flow patterns at certain heights but fail, due to their simplifications, in replicating the entire three-dimensional flow structure obtained experimentally.

1. Introduction

Within the Wind Engineering community, increasing attention is being paid to the effects of non-stationary, non-synoptic winds, i.e. tornadoes. The structure of full-scale tornadoes is highly complex, showing a three-dimensional flow field, instabilities, singularities and non-linear effects (e.g. Lewellen, 1993; Davies-Jones et al., 2001; Alexander and Wurman, 2008; Karstens et al., 2010). In order to understand the physical processes present in a tornado flow field, simplified models are needed, which reduce the degree of freedom present in full-scale observations, and therefore allow a detailed and statistically representative evaluation of velocity and pressure fields. In order to provide this type of datasets, the tornado-like flow field was modelled experimentally and/or numerically by several authors such as Ward, 1972; Church et al., 1979; Lewellen et al., 1997; Haan et al., 2008; Mishra et al., 2008; Natarajan, 2011; Sabareesh et al., 2012; Refan et al., 2014; Gillmeier et al., 2016; Liu and Ishihara, 2016; Nolan et al., 2017 and Tang et al., 2017. An attempt to analytically model the three-dimensional flow in the boundary layer of a tornado-like vortex was made by Kuo (1971) by alternatingly solving the two nonlinear boundary-layer equations for the radial and vertical distribution of velocities. The Bloor and Ingham vortex model (1987) and the Vyas-Majdalani vortex model (Vyas et al., 2003) are exact inviscid solutions to the Euler's equations in a confined conical and cylindrical domain, respectively. Xu and Hangan (2009) analytically modelled an inviscid tornado-like vortex using a free narrow jet solution combined with a modified Rankine vortex. However, it needs to be mentioned here that this combined model is not an exact solution to the Navier-Stokes-Equations. Wood and White (2011) presented a new parametric model of vortex tangential-wind profiles, which is based on the Vatistas model (Vatistas et al., 1991) and is primarily designed to depict realistic-looking tangential wind profiles observed in atmospheric vortices.

Despite this excellent work, the Rankine (1882), Burgers-Rott (Burgers, 1948; Rott, 1958) and Sullivan (1959) vortex model are still the most commonly used vortex models to replicate tornado-like flow behaviour. An overview of some of the before mentioned vortex models can be found in e.g. Kilty (2005), Batterson et al. (2007) and Kim and Matsui (2017). However, with the increasing interest in the simulation of tornado-like flows, these models have (in some cases and with varying degrees of success) been invoked in order to describe some elements of the flow field. The authors feel that it is worth reflecting on the fundamental assumptions behind these models and bench marking their performance against measured data obtained in controlled conditions. For that reason, this paper gives detailed information about the derivation and simplifications of the Rankine, Burgers-Rott and Sullivan vortex model. In addition to the Rankine, Burgers-Rott and Sullivan vortex

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models, the recently published vortex model by Baker and Sterling (2017), hereafter called 'Baker vortex model', is also included in the analysis.

Section two of this paper provides detailed information about the derivation and simplifications of the above mentioned vortex models, while section three outlines the experimental methodology used to assess the model suitability. The results of the model benchmarking can be found in section four, with the main conclusions presented in section five.

2. Existing vortex models

2.1. Flow field notation

In what follows, a cylindrical coordinate system has been adopted as illustrated in Fig. 1. In Fig. 1, r, z and θ are the radial distance, vertical distance and circumferential angle, respectively. Thus u_r, u_z and u_θ represent the radial, vertical and circumferential components of velocity. For the sake of simplicity the flow is considered to be incompressible for all models and a density of air of $\rho = 1.21\ \mathrm{kg/m^3}$ is assumed for all calculations. In this section, a brief description of the different vortex models examined in this paper is provided, together with the underlying assumptions.

Using the aforementioned notation, the continuity equation (Eq. (1)) and radial (Eq. (2)), circumferential (Eq. (3)) and vertical (Eq. (4)) components of the Navier-Stokes-Equations (NSE) can be expressed as:

$$\underbrace{\frac{1}{r}\frac{\partial(ru_r)}{\partial r}}_{1}+\underbrace{\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}}_{2}+\underbrace{\frac{\partial u_z}{\partial z}}_{3}=0 \tag{1}$$

$$\underbrace{\frac{\partial u_r}{\partial t}}_{R1}+\underbrace{u_r\frac{\partial u_r}{\partial r}}_{R2}+\underbrace{\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}}_{R3}-\underbrace{\frac{u_\theta^2}{r}}_{R4}+\underbrace{u_z\frac{\partial u_r}{\partial z}}_{R5}$$
$$=-\underbrace{\frac{1}{\rho}\frac{\partial p}{\partial r}}_{R6}+\underbrace{g_r}_{R7}+\nu\left(\underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right)}_{R8}-\underbrace{\frac{u_r}{r^2}}_{R9}+\underbrace{\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}}_{R10}\right.$$
$$\left.-\underbrace{\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}}_{R11}+\underbrace{\frac{\partial^2 u_r}{\partial z^2}}_{R12}\right) \tag{2}$$

$$\underbrace{\frac{\partial u_\theta}{\partial t}}_{C1}+\underbrace{u_r\frac{\partial u_\theta}{\partial r}}_{C2}+\underbrace{\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}}_{C3}+\underbrace{\frac{u_r u_\theta}{r}}_{C4}+\underbrace{u_z\frac{\partial u_\theta}{\partial z}}_{C5}$$
$$=-\underbrace{\frac{1}{\rho r}\frac{\partial p}{\partial \theta}}_{C6}+\underbrace{g_\theta}_{C7}+\nu\left(\underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right)}_{C8}-\underbrace{\frac{u_\theta}{r^2}}_{C9}+\underbrace{\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}}_{C10}\right.$$
$$\left.-\underbrace{\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}}_{C11}+\underbrace{\frac{\partial^2 u_\theta}{\partial z^2}}_{C12}\right) \tag{3}$$

$$\underbrace{\frac{\partial u_z}{\partial t}}_{Z1}+\underbrace{u_r\frac{\partial u_z}{\partial r}}_{Z2}+\underbrace{\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}}_{Z3}+\underbrace{u_z\frac{\partial u_z}{\partial z}}_{Z4}$$
$$=-\underbrace{\frac{1}{\rho}\frac{\partial p}{\partial z}}_{Z5}+\underbrace{g_z}_{Z6}+\nu\left(\underbrace{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)}_{Z7}+\underbrace{\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}}_{Z8}+\underbrace{\frac{\partial^2 u_z}{\partial z^2}}_{Z9}\right) \tag{4}$$

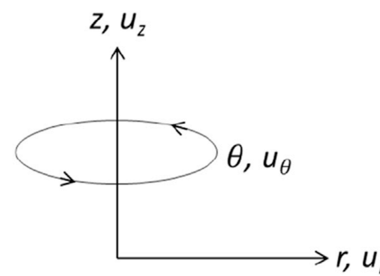

Fig. 1. Flow field notation.

Here ρ is the density of the fluid, t is the time, p is the static pressure and $\vec{g}$ is the gravity vector in its different components. The different terms in equations (1)–(4) have been labelled since, as will be demonstrated below, it is possible to derive the majority of the analytical models by disregarding different terms.

2.2. Rankine vortex model

The Rankine model has been adopted by a number of researchers (e.g. Hoecker, 1960; Church et al., 1979; Winn et al., 1999; Wurman and Gill, 2000; Brown and Wood, 2004; Lee et al., 2004; Mishra et al., 2008; Bech et al., 2009; Hashemi Tari et al., 2010; Wood and Brown, 2011; Refan and Hangan, 2016; Tang et al., 2017) to model tornado-like flow behaviour. The following assumptions are made in the derivation of the Rankine vortex model:

- *The flow field is one-dimensional and as such* equations (2) and (4) *can be disregarded.*
- *The flow field is steady state, i.e., term R1 can be taken as zero.*
- *The flow is inviscid (μ=0), i.e., terms R8 - R12 can be neglected.*
- *Body forces can be neglected, i.e., ($\vec{g}$=0).*

These assumptions reduce the NSE to the cyclostrophic equation (Eq. (5)).

$$\frac{dp(r)}{dr}=\rho\,\frac{u_\theta(r)^2}{r} \tag{5}$$

The Rankine model also assumes that the flow consists of two separate flow regions. In the first region, the core region (i.e., r R, where R is the core radius, which is defined as the radial distance from the vortex centre at which the circumferential velocity component is maximal), the flow is assumed to have a constant vorticity and is considered to be similar to that of a solid body. In the second region, ($r > R$) it is assumed that the flow can be described by a potential flow field (incompressible, inviscid and irrotational) (Alekseenko et al., 2007) and is inversely proportional to the radial distance. These assumptions enable the circumferential velocity component to be modelled via an expression of the form:

$$\overline{u_\theta}(\overline{r})=\begin{cases}\overline{r} & for\ (\overline{r}<1)\\ \dfrac{1}{\overline{r}} & for(\overline{r}>1)\end{cases} \tag{6}$$

where $\overline{u_\theta}$ is the normalised circumferential velocity component ($= u_\theta/u_{\theta,max}$, where $u_{\theta,max}$ is the maximum value of u_θ) and $\overline{r}$ is the radial distance normalised by the core radius, R. In equation (6), a discontinuity occurs at $\overline{r}=1$. In order to avoid this, the model is occasionally modified as shown in equation (6.1). However, the most commonly used form is shown in equation (6) and hence will be used in what follows.

$$\overline{u_\theta}(\overline{r})=\frac{2\overline{r}}{(1+\overline{r}^2)} \tag{6.1}$$