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Research paper

An interval multi-objective programming model for irrigation water allocation under uncertainty

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ABSTRACT

An interval linear multi-objective programming (ILMP) model for irrigation water allocation was developed, considering conflicting objectives and uncertainties. Based on the generation of interval numbers through statistical simulation, the ILMP model was solved using a fuzzy programming method. The model balances contradictions among economic net benefit, crop yield and water-saving in irrigation systems incorporating uncertainties in both objective functions and constraints that are based on the conjunctive use of surface water and groundwater. The model was applied to Hulan River irrigation district, northeast China. Tradeoffs between various crops in different subareas under different frequencies were analyzed, and scenarios with different objectives were considered to evaluate the changing trend of irrigation water allocation.Results indicated that the ILMP model provided effective linkages between revenue/output promotion and water saving, and offers insights into tradeoffs for irrigation water management under uncertainty.

1. Introduction

These days water crisis is occurring too frequently and at too many places, underscoring the importance of sustainable and efficient water resources management. Among all the water users, agriculture is the dominant user and irrigated agriculture consumes more than 70% of available water resources in the world (Galán-Martín et al., 2017). Due to rapid socio-economic development and continuing population growth, future irrigated agriculture will face challenges to meet the growing food demand, while the water available for agriculture will simultaneously be decreasing (Wang et al., 2017). Therefore, optimal allocation of water availability for agricultural irrigation in an efficient manner is a critical issue for agricultural water management.

Optimal allocation of agricultural irrigation water can be determined using optimization techniques. A number of optimization techniques, such as linear programming, dynamic programming, nonlinear programming, and stochastic programming, have been employed to drive optimal irrigation patterns subject to the maximization or minimization of certain objectives (Singh, 2012). Among these optimization techniques, linear programming for irrigation water allocation has been most popular (Das et al., 2015). However, irrigation water allocation systems depend on various independent aspects, such as economic, social and natural, that often conflict with each other, and are therefore too complex for linear programming. Such a water allocation problem can be handled by multi-objective programming (MOP) that is capable of incorporating multiple conflicting objectives functions, such as maximizing net return/crop yield versus reducing water consumption. Some investigators have used MOP for allocating irrigation water with a different emphasis. For example, Su et al. (2014) developed a multi-objective optimal allocation model for agricultural water resources considering three objectives of the maximum net benefit from agriculture, the minimum fairness difference in the utilization of water, and the maximum proportion of green water utilization. Galán-Martín et al. (2017) formulated a multi-objective linear programming model that simultaneously accounts for the maximization of crop production and the minimization of environmental impact caused by water consumption.

However, optimal allocation of irrigation water in real world is complicated by various uncertainties that arise in the interac-

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tions among many system components. Examples include temporal and spatial variations of hydrological elements, fluctuations of economic parameters, and errors in estimating other related parameters. Therefore, it is essential to optimize irrigation water allocation in the framework of MOP considering uncertainties in order to better represent the complexity of irrigation systems. Generally, the widely used uncertainty methods are stochastic mathematical programming (SMP), fuzzy mathematical programming (FMP), and interval mathematical programming (IMP). SMP is an optimization model wherein parameters in the objective functions or constraints can be represented by probability distributions. FMP addresses vagueness in decision maker' aspirations (or preference) and ambiguity in knowledge or information in an optimization model. IMP deals with uncertainties that are approximated by only the lower and upper boundaries. These uncertainty methods can handle different types of uncertainties, but some deficiencies also exist. For example, although SMP is capable for adequately tackling uncertainties but the high computations data requirement for specifying probability distributions may affect its practical application. For the FMP, there exists subjectivity in the membership function determination and results generation. Recently, some investigators focused on fuzzy uncertainties in the MOP model for irrigation water allocation (Regulwar and Gurav, 2011; Mirajkar and Patel, 2013; Li and Guo, 2014; Morankar et al., 2016; Li et al., 2016a,b). In practice, specification of fuzzy sets or probability distributions is more difficult than obtaining interval numbers, especially in the absence of data. For an irrigation system, many basic data are obtained by field experiments and monitoring, resulting in the time series of these primary data are fairly short. Therefore, considering the availability of data and computational efficiency, the use of interval numbers is particularly appealing for an irrigation system compared with SMP and FMP although the IMP may encounter difficulties in tackling higher uncertain parameters. Thus far, few investigations have considered interval uncertainties in MOP for irrigation water allocation.

Because of the capability to consider uncertainties with known lower and upper bounds in both objective functions and constraints, IMP has been successfully integrated in several single objective programming models for irrigation water allocation (Lu et al., 2011; Li et al., 2014; Guo et al., 2014; Yang et al., 2015, 2016). However, few studies provide details on methods for acquisition of interval numbers in the optimization of irrigation water allocation. Hydrological elements, such as runoff, precipitation, and evapotranspiration, directly affect optimal irrigation water allocation. These elements spatially and temporally change and can essentially be regarded as random parameters. Considerable differences occur in the quantities of these hydrological elements for different frequencies. Determining the fluctuation values of these elements for each frequency in advance can not only benefit irrigation optimization but also help ameliorate natural disasters, such as flooding and drought. Therefore, generation of interval numbers of hydrological elements for different frequencies that are incorporated in the MOP optimization model can be valuable not only for avoiding sophisticated calculations but also for precision irrigation.

The primary objective of this study therefore is to develop an interval linear multi-objective programming (ILMP) model for optimal irrigation water allocation under uncertainty. The model incorporates the optimization technique of interval parameters into linear multi-objective programming to handle uncertainties of irrigation systems and achieves a balance among net revenue, output, and water-saving by optimally allocating available water. The study entails several elements. First, interval numbers for social-economic data and hydrological elements for different frequencies are generated; second, the ILMP model is developed for irrigation water allocation, based on the generation of interval parameters; third, results are analyzed for different hydrological frequencies; fourth, a scenarios analysis is done as a complementary method to develop more decision-making plans under different policy conditions; and the model is tested by applying to a real-world study in northeast China.

2. Methodology

This section develops an interval linear multi-objective programming (ILMP) framework, with emphasis on the (1) generation of interval parameters using bootstrap method; (2) development of the ILMP model based on the generation of interval parameters; (3) solution of the ILMP model based on fuzzy programming (FP) method; and (4) development of a framework for multi-objective programming under uncertainty. Each of these elements is now discussed.

2.1. Bootstrap method

The bootstrap method is a resampling technique. This method only needs resampling from the original sample series without making assumptions for the overall distribution, then continually estimates parameter values of the drawn samples, and finally deduces the parameter characteristics of the unknown overall samples and quantitatively describes the uncertainty of parameter estimation. Thus, it is an effective way to estimate and generate interval numbers based on long-term data.

Assuming $X = (x_1, x_2, \cdots, x_n)$ is an original sample and θ is the unknown parameter of the overall distribution. From the lowest value to the highest value, the original sample can be written as $x_{(1)} \le x_{(2)} \le \cdots \le x_{(n)}$. The empirical distribution F_n for the sample can be described as follows (Hu et al., 2015):

$$F_n = \begin{cases} 0, & x \ll x_{(1)} \\ \dfrac{k}{n}, & x_{(k)} \le x \ll x_{(k+1)}, k = 1, 2, \cdots, n-1 \\ 1, & x \ge x_{(n)} \end{cases} \tag{1}$$

By resampling from the distribution F_{n}, the same size sample $X^* = \left(x_1^*, x_2^*, \cdots, x_n^*\right)$ can be obtained. Based on the bootstrap sample X^*, the estimation θ^* of parameter θ of the distribution function can be calculated by a proper parameter estimation method. Repeating the bootstrap sampling N times, the N groups of bootstrap samples can be obtained and can be described as $X^{*(j)} = \left(x_1^{*(j)}, x_2^{*(j)}, \cdots, x_n^{*(j)}\right)$ $(j = 1, 2, \cdots, N)$. Then N parameter estimates $\theta^{*(j)}$ $(j = 1, 2, \cdots, N)$ of parameter θ can be derived. Taking $\theta^{*(j)}$ $(j = 1, 2, \cdots, N)$ as the sample of the unknown parameter θ, the distribution of parameter θ can thus be obtained. Based on this distribution, interval estimation of the values under each frequency can be derived based on the confidence intervals under predetermined confidence levels.

2.2. Interval linear multi-objective programming

2.2.1. Property of the interval numbers

Before formulating the ILMP model, it is appropriate to first discuss the properties of interval numbers.

Property 1: Let A denote a closed and bounded set of real numbers, and $A^{\pm}$ define an interval number with known upper and lower bounds of A. Then $A^{\pm}$ can be expressed as$A^{\pm} = [A^-, A^+] = \left\{A^- + z(A^+ - A^-) | 0 \le z \le 1\right\}$, with A^- and A^+ representing the lower and upper bounds of interval number $A^{\pm}$, respectively, and z representing an auxiliary variable that can be used to transform the interval parameter into a determination one.

Property 2: For $A^{\pm}$, the following relationships hold: (1) $A^{\pm} \ge 0$ if $A^- \ge 0$ and $A^+ \gg> 0$; and (2) $A^{\pm} \ll 0$ if $A^- \le 0$ and $A^+ \le 0$. Further, for