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Interval semantics for Petri nets with inhibitor arcs

Mohammad Alqarni [a], Ryszard Janicki [b,*]

[a] *Faculty of Computing and Information Technology, University of Jeddah, Jeddah, Saudi Arabia*
[b] *Department of Computing and Software, McMaster University, Hamilton, ON, L8S 4K1 Canada*

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ABSTRACT

Interval semantics of elementary Petri nets with inhibitor arcs is discussed. First an operational semantics in terms of *interval orders* is provided, and next the concept of *interval process* is introduced, discussed, and used to describe concurrent histories of such nets. It is shown that the interval process semantics is equivalent to recently proposed *interval traces* semantics. It is also proven that if operational semantics is restricted to stratified orders (i.e. step sequences) the proposed model is equivalent to models based on step processes and comtraces.

1. Introduction

Concurrent systems are complex in nature as they tend to produce behaviours that involve a number of distributed components which run concurrently, need to communicate, and synchronize. Due to this complexity, modeling concurrent systems have undergone extensive research from which a number of mathematical modeling languages have emerged.

Standard operational semantics of the majority of concurrency models are defined in terms of either firing sequences or firing step sequences, while standard concurrent history semantics is usually defined in terms of partial orders, or stratified order structures (or structures equivalent to them such as traces or net processes), depending on assumptions made about observing system runs.

Nevertheless, it is commonly assumed (first argued by N. Wiener in 1914 [47] and analysed in detail in [22]) that any system run (execution) that can be observed by a single observer *must* be an interval order of event occurrences. This means that the most precise observational semantics is defined in terms of interval orders. Moreover, representing observations as interval orders allows the capturing of behaviours that neither of the standard semantics can really describe. However generating interval orders directly is problematic for most models of concurrency, as the only feasible *sequence representation* of interval order is by using Fishburn Theorem [12] and appropriate sequences of *beginnings* and *endings* of events involved, which makes modeling much more complex and difficult [22,25,36,42,44].

Elementary Petri nets with inhibitor arcs [8,23] are very simple. They are just classical elementary nets of [35,40] extended with inhibitor arcs. However, they can model extremely complicated behaviours [5,23,28], that cannot easily (or not at all) be represented by other models. For example they can easily model the "not later than" relationship, which cannot be represented by classical place/transition nets [27,39] and most other popular models. Hence, the elementary nets with inhibitor arcs are an excellent medium for novel models of behaviours.

* Corresponding author.
E-mail addresses: maaalqarni8@uj.edu.sa (M. Alqarni), janicki@mcmaster.ca (R. Janicki).

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While validity of operational semantics is usually obvious, the validity of concurrent history/behaviour semantics is often not. It relies on the validity of the definition of a concurrent history/behaviour, which is often not trivial and may involve complex reasoning (cf. [5,19,20,35]). On the other hand, the *process semantics* (in the sense of [7,29,35,40]), *does not usually require much validation as intuitively it is just a set of system unfoldings, so it is as natural as any operational semantics* (cf. [28,35,45]). Hence, it can be used as a benchmark for validity of other types of history/behaviour semantics, they just have to be equivalent to the process semantics (cf. [7,29]). For Petri nets with inhibitor arcs and step sequence operational semantics, an elegant stratified process semantics has been proposed in [28].

In this paper we propose an interval process semantics for Petri nets with inhibitor arcs under the assumption that operational semantics is defined by interval orders and the interval orders are represented by appropriate sequences of their beginnings and endings.

It was shown in [24] that concurrent histories involving observations represented by interval orders can be represented by *interval order structures* (proposed in [21,30]), but how these interval order structures could be derived for particular concurrent systems is not clear. *We will show that our interval processes can be interpreted as appropriate interval order structures.*

Behaviours of concurrent systems can often be elegantly modeled using *traces*. When concurrent histories are fully described by causality relations, i.e. partial orders, we may use *Mazurkiewicz traces* [10,31,32], when concurrent histories are represented by stratified order structures we may use *comtraces* [23], and when concurrent histories are represented by interval order structures we may use recently introduced *interval traces* [25]. *We will show that for a given elementary Petri net, each process can be transformed into an appropriate interval trace, and they both represent exactly the same interval order structure, i.e. a concurrent history.*

Various process semantics for elementary Petri nets with inhibitor arcs that started from various assumptions have been proposed in the past, the most representative and prolific are probably [7,23,27,28,33,45,48], but *they all assume that the operational semantics is defined in terms of sequences* [33,45,48] *or step sequences* [7,23,27,28,36]. None of these models is able to deal with observations (system runs) that are neither step sequences nor semantically equivalent to any step sequence, as for example interval orders. *We will show that our interval model covers, and is consistent with, the models where sequences or step sequences were used to represent system runs.*

Some initial preliminary results have been published in [4]. However, the conference paper [4] lacks in containing full proofs, has a limited explanation of the intuition behind many constructions, and most importantly, it does not consider the relationship to interval trace semantics. This paper contains full proofs, a detailed relationship to the model of [28], a better explanation of intuitions, and also the proof that our interval process semantics and *interval trace* semantics of [25] are equivalent.

The paper is organized as follows. Section 2 provides standard mathematical definitions for different partial orders and their sequence representations. In Section 3, stratified and interval order structures are briefly discussed. An overview of elementary Petri nets with inhibitor arcs and their operational semantics is presented in Section 4, while elementary Petri nets with activator arcs are briefly discussed in Section 5. A simple motivational example that provides some intuitions and illustrates the main concepts of this paper is analyzed in details in Section 6. Figs. 4 and 5 show net representation of our running example and illustrate the intuition of our approach. Interval elementary nets with inhibitor arcs, our basic research medium, are discussed in Section 7. Section 8 is devoted to the process semantics for interval elementary nets with inhibitor arcs, as proposed in [28]. The first main contribution of this paper, namely, the concept of interval process and the relationship between interval processes and interval order structures, forms Section 9. In Section 10 we show that when system runs are restricted to step sequences our model is the same as that of [28]. Interval trace semantics of elementary nets with inhibitor of [25] is briefly recalled in Section 10. In Section 12 we will prove that our interval process semantics and *interval trace* semantics of [25] are equivalent, which is the second main contribution of this paper. Section 13 contains final comments and conclusions.

2. Partial, total, stratified and interval orders, and sequence representations

We will start with a short introduction to partial orders (cf. [13]), as they are the principal tool used to describe executions and operational semantics of concurrent systems.

Definition 1. A relation $<\,\subseteq X \times X$ is a (*strict*) ***partial order*** iff it is irreflexive and transitive, i.e. for all $a, c, b \in X$, $a \not< a$ and $a < b < c \implies a < c$. We also define:

$$a \frown_< b \stackrel{df}{\iff} \neg(a < b) \wedge \neg(b < a) \wedge a \neq b, \text{ and}$$
$$a <^\frown b \stackrel{df}{\iff} a < b \vee a \frown_< b.$$

Note that $a \frown_< b$ means a and b are *incomparable* (w.r.t. $<$) elements of X. ■

Let $<$ be a partial order on a set X. Then:

1. $<$ is ***total*** if $\frown_< = \emptyset$. In other words, for all $a, b \in X$, $a < b \ \vee\ b < a \ \vee\ a = b$;
2. $<$ is ***stratified*** if $a \frown_< b \frown_< c \implies a \frown_< c \vee a = c$, i.e., the relation $\frown_< \cup\, id_X$ is an equivalence relation on X;
3. $<$ is ***interval*** if for all $a, b, c, d \in X$, $a < c \wedge b < d \implies a < d \vee b < c$. In other words, $<$ is interval if all its four element restrictions are different from $<_4$ in Fig. 1.