Information and Computation 253 (2017) 78–108

Contents lists available at ScienceDirect

Information and Computation

www.elsevier.com/locate/yinco

Modeling concurrency with interval traces

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ARTICLE INFO

Article history:
Received 29 September 2014
Received in revised form 14 December 2015
Available online 2 January 2017

ABSTRACT

Interval order structures are useful tools to model abstract concurrent histories, i.e. sets of equivalent system runs, when system runs are modeled with *interval orders*. This paper shows how interval order structures can be modeled by *partially commutative monoids*, called *interval traces*. The model is then used to provide a semantics of Petri nets with inhibitor arcs, both in terms of interval traces and in terms of interval order structures.

1. Introduction

Most observational[1] semantics of concurrent systems are defined either in terms of sequences (i.e. total orders) or step-sequences (i.e. stratified orders). When concurrent histories[2] are fully described by *causality relations*, i.e. *partial orders*, Mazurkiewicz traces [10,33,34] allow a representation of the entire partial order by a single sequence (plus *independency* relation), which provides a simple and elegant connection between observational and process semantics (i.e. the semantics in terms of concurrent histories) of concurrent systems with static concurrency structure, i.e. if two actions are independent, they are always independent. In such case, all other relevant observations can be derived as just stratified or interval extensions of appropriate partial orders.

It has been observed a long time ago that if priority and concurrency are mixed, it may happen that for two actions a and b, a sequence a followed by b, and a simultaneous execution of a and b are allowed, and they can be considered as equivalent, but the sequence b followed by a is disallowed [16,30,43]. Such situation is often called "not later than" relationship (cf. [17,28]) as a may not follow b (but the opposite order and simultaneity are allowed).

When we want to model both causality and the "not later than" relationship, we have to use *stratified order structures* [14,20,22], when *all* observations are step-sequences, or *interval order structures* [23,20,31], when *all* observations are interval orders.

Comtraces [22] allow a representation of stratified order structures by single step-sequences (with appropriate *simultaneity* and *serializability* relations).

It was argued by Wiener in 1914 [51] (and later more formally in [21]) that any execution that can be observed by a single observer must be an interval order. It implies that the most precise observational semantics is defined in terms of

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[1] 'Observational semantics' is not a generally agreed concept, in this paper this will be just a collection of all system runs (i.e. executions, observations) [5,21,28,49]. A different meaning is used in for example [7].

[2] In this paper a 'concurrent history' is a set of equivalent system runs (executions, observations), represented uniquely by some partial order or order structure, and differs formally (although it is intuitively close) to these of for instance [8,26]. The concept used in this paper was introduced in [21] and is close to that of [34].

http://dx.doi.org/10.1016/j.ic.2016.12.009
0890-5401/

interval orders. However generating interval orders directly is problematic for most models of concurrency. Unfortunately, the only feasible sequence representation of interval order is by using sequences of *beginnings* and *endings* of events involved [11,21].

The goal of this paper is to provide a monoid based model that allows a single sequence of beginnings and endings (enriched with appropriate *simultaneity* and *serializability* relations) to represent the entire *stratified order structures* as well as all equivalent interval order observations. This will be done by introducing and developing the concept of *interval traces*, a mixture of ideas from both Mazurkiewicz traces [10] and representations of interval orders [12], and proving that each interval trace uniquely determines an interval order structure. The interval traces considered in this paper are highly revised, modified and extended version of the concept originally proposed in [25].

We will also show how interval traces can define interval order semantics of elementary nets with inhibitor arcs.

Modeling observational semantics with sequence and concurrent histories with Mazurkiewicz traces (cf. [10]) as well as modeling observational semantics with step sequence and concurrent histories with comtraces (or similar models, cf. [5,19,22,24,28,32,49]) is well developed and relatively well known. Recently published [19] provides a general model that covers most others as special cases, including these from [5,22,24,49]). For the case where the system runs or observations are represented by intervals or interval orders, the situation is much less impressive [21,25,40,45–49]. Conceptually closest to our approach are models based on the concept of ST-traces (sequences of transition beginnings and ends) and ST-bisimulation [45–49]. We will briefly discuss some relationships of ST-traces to our model in Section 10.5. The beginnings and ends are also used in [40], but the outcome is only step sequence semantics. The paper [21] provides some general abstract results and [25] defines an initial version of interval traces and some preliminary results.

This paper is organized as follows. Section 2 recalls some concepts and results on partial orders, sequences, and their mutual relationship. In Section 3, Mazurkiewicz traces and their basic properties are briefly discussed. Interval traces, the main concept introduced in this paper, are discussed in Section 4. Section 5 is devoted to interval order structures and their partial order representations. The relationship between interval traces and interval order structures, one of the main contributions of this paper, is discussed in Section 6, and the relationship between interval traces and comtraces in Section 7. The concept of concurrent histories and how they relate to interval traces is analyzed in Section 8. In Section 9, it is shown how interval traces can describe various behavioral properties of concurrent systems, and a small example is analyzed in detail. In Section 10 we show how interval traces can be used to provide an adequate semantics of elementary Petri nets with inhibitor arcs. The relationship to the model of [49] is also discussed at the end of Section 10. Section 11 contains some final comments. Technical proofs of two results are given in Appendix A.

2. Partial orders and sequences

In this section, we recall some, often well-known, concepts, notations and results regarding partial orders [12], sequences and representations of partial orders by appropriate sequences [13,18,22,24].

2.1. Partial orders

Partial orders are one of the basic tools used in this paper. They will be used as a full representation of systems runs (or observations) and as a partial representation of concurrent histories.

Definition 1. A relation $<\subseteq X\times X$ is a (*strict*) *partial order* iff it is irreflexive and transitive, i.e. for all $a,c,b\in X$, $a\not< a$ and $a<b<c \implies a<c$. We also define:

$$a \frown_< b \overset{df}{\iff} \neg(a<b)\wedge\neg(b<a)\wedge a\neq b,$$

$$a <^\frown b \overset{df}{\iff} a<b \vee a\frown_< b.$$

Note that $a\frown_< b$ means a and b are *incomparable* (w.r.t. $<$) elements of X. ■

Let $<$ be a partial order on a set X. Then:

1. $<$ is *total* if $\frown_<=\emptyset$. In other words, for all $a,b\in X$, $a<b \vee b<a \vee a=b$. For clarity, we will reserve the symbol $\lhd$ to denote total orders;
2. $<$ is *stratified* if $a\frown_< b\frown_< c \implies a\frown_< c\vee a=c$, i.e., the relation $\frown_< \cup\, id_X$ is an equivalence relation on X;
3. $<$ is *interval* if for all $a,b,c,d\in X$, $a<c \wedge b<d \implies a<d\vee b<c$.

It is clear from these definitions that every total order is stratified and every stratified order is interval. An interval order is *strict* if it is not stratified. In this paper, most partial orders will be represented by Hasse diagrams [12]. The following simple concept will often be used in this paper.

Definition 2. For a relation $R\subseteq X\times X$, any relation $Q\subseteq X\times X$ is an *extension* of R if $R\subseteq Q$. ■