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Trace inclusion for one-counter nets revisited

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ABSTRACT

One-counter nets (OCN) consist of a nondeterministic finite control and a single integer counter that cannot be fully tested for zero. They form a natural subclass of both One-Counter Automata, which allow zero-tests and Petri Nets/VASS, which allow multiple such weak counters. The trace inclusion problem has recently been shown to be undecidable for OCN. In this paper, we contrast the complexity of two natural restrictions which imply decidability.
We show that trace inclusion between a OCN and a *deterministic* OCN is NL-complete, even with arbitrary binary-encoded initial counter-values as part of the input. Secondly, we show that the trace *universality* problem of nondeterministic OCN, which is equivalent to checking trace inclusion between a finite system and a OCN-process, is Ackermann-complete.

1. Introduction

A fundamental question in formal verification is if the behavior of one process can be reproduced by – or equals that of – another given process. These inclusion and equivalence problems, respectively have been studied for various notions of behavioral preorders and equivalences and for many computational models. Trace inclusion/equivalence asks if the set of *traces*, all emittable sequences of actions, of one process is contained in/equal to that of another. Other than for instance simulation preorder, trace inclusion lacks a strong locality of failures, which makes this problem intractable or even undecidable already for very limited models of computation.

We consider one-counter nets, which consist of a finite control and a single integer counter that cannot be fully tested for zero, in the sense that an empty counter can only restrict possible moves. They are subsumed by One-counter automata (OCA) and thus Pushdown Systems, which allow explicit zero-tests by reading a bottom marker on the stack. At the same time, OCN are a subclass of Petri Nets or Vector Addition Systems with states (VASS): they are exactly the one-dimensional VASS and thus equivalent to Petri Nets with at most one unbounded place.

Related work. Valiant and Paterson [1] show the decidability of the trace equivalence problem for *deterministic* one-counter automata (DOCA). This problem has subsequently been shown to be NL-complete by Böhm, Göller and Jančar [2], assuming fixed initial counter-values. The equivalence of deterministic pushdown automata is known to be decidable [3] and primitive recursive [4], but the exact complexity is still open.

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Valiant [5] proves the undecidability of both trace inclusion for DOCA and universality for nondeterministic OCA. Jančar, Esparza and Moller [6] consider trace inclusion between Petri Nets and finite systems and prove decidability in both directions. Jančar [7] showed that trace inclusion becomes undecidable if one compares processes of Petri Nets with at least two unbounded places. In [8], the authors show that trace inclusion is undecidable already for (nondeterministic) one-counter nets. Simulation preorder however, is known to be decidable and PSPACE-complete for this model [9–11], which implies a PSPACE upper bound for trace inclusion on DOCN as both relations coincide for deterministic systems.

Higuchi, Tomita and Wakatsuki [12,13] compare the classes of *languages* defined by DOCN with various acceptance modes and in a series of papers consider the respective inclusion problems. They derive procedures that exhaustively search for a bounded witness that work in time and space polynomial in the size of the automata if the initial counter-values are fixed. We show that for monotone relations like trace inclusion or the inclusion of languages defined by acceptance with final states, one can speed up the search for suitable witnesses.

Our contribution. We fix the complexity of two well-known decidable decision problems regarding the traces of one-counter processes. We show that trace inclusion between *deterministic* OCNs is NL-complete. Our upper bound holds even if only the supposedly larger process is deterministic and if (binary encoded) initial counter-values are part of the input. Technically, we show a small witness property for the existence of (possibly long) distinguishing traces. Our certificates are similar to the linear path schemes in [14] and our characterization can be interpreted as showing that the 2-dim. product automaton is flattable in the sense of [15]. The sizes of certificates is polynomial in the number of states of the finite control and they can be verified in space logarithmic in the binary representation of the initial counter-values.

Our second result is that trace universality of *nondeterministic* OCN is Ackermann-complete. This problem is (logspace) inter-reducible with checking trace inclusion between a finite process and a process of a OCN.

2. Background

We write $\mathbb{N}$ for the set of non-negative integers. For any set A, let A^* denote the set of finite strings over A and $\varepsilon \in A^*$ the empty string.

Definition 1. A *one-counter automaton* $\mathcal{A} = (Q, Act, \delta, \delta_0)$ is given by finite sets of control states Q, action labels Act, transitions $\delta \subseteq Q \times Act \times \{-1, 0, 1\} \times Q$ and zero-test transitions $\delta_0 \subseteq Q \times Act \times \{0, 1\} \times Q$. It induces an infinite-state labeled transition system over the state set $Q \times \mathbb{N}$, whose elements will be called *processes* and written as pm where $p \in Q$ and $m \in \mathbb{N}$. The transition relation $\longrightarrow \; = \; \longrightarrow_+ \cup \longrightarrow_0$ is partitioned into *positive* and *zero-testing* steps. For states $p, p' \in Q$ and $m, m' \in \mathbb{N}$ these are defined by

1. $pm \xrightarrow{a}_+ p'm' \iff (p, a, (m' - m), p') \in \delta$ and
2. $pm \xrightarrow{a}_0 p'm' \iff (p, a, m', p') \in \delta_0$ and $m = 0$.

Such an automaton is called a *one-counter net* if $\delta_0 = \emptyset$, i.e., if the automaton cannot test if the counter is equal to 0.

When defining a OCN we will omit δ_0 and simply define the net as triple $\mathcal{N} = (Q, Act, \delta)$. Abusing notation, we moreover write $pm \xrightarrow{t} qn$ if a transition $t = (p, a, d, q) \in \delta \cup \delta_0$ justifies a step $pm \xrightarrow{a} qn$.

Definition 2 *(Traces).* Let pm be a process of the OCN $\mathcal{N}$. The *traces* of pm are the elements of the set

$$T_{\mathcal{N}}(pm) = \{a_1 a_2 \ldots a_k \in Act^* \mid \exists qn \; pm \xrightarrow{a_0} \circ \xrightarrow{a_1} \circ \cdots \circ \xrightarrow{a_k} qn\}.$$

We will omit the index $\mathcal{N}$ if is clear from the context. *Trace inclusion* is the decision problem that asks if $T_{\mathcal{A}}(pm) \subseteq T_{\mathcal{B}}(p'm')$ holds for given processes pm and $p'm'$ of nets $\mathcal{A}$ and $\mathcal{B}$, respectively. *Trace universality* asks if $Act^* \subseteq T(pm)$ holds for a given process pm.

An important property of one-counter nets is that the step relation and therefore also trace inclusion is monotone with respect to the counter:

Lemma 1 *(Monotonicity). If $pm \xrightarrow{a} p'm'$ then $p(m+1) \xrightarrow{a} p'(m'+1)$. This in particular means that $T(pm) \subseteq T(p(m+1))$ holds for any OCN-process pm.*

Remark 1. In this paper we consider what are sometimes called *realtime* automata, in which no silent (ε-labeled) transitions are present. This is no restriction: the usual syntactic requirement for DPDA, that no state with outgoing ε-transition may have outgoing transitions labeled by $a \neq \varepsilon$, together with the monotonicity of steps in OCN, implies that all states on ε-cycles are essentially deadlocks. One can thus eliminate ε-labeled transitions in logarithmic space.

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