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A Mixed-Boolean Hybrid Mathematical Model with Discontinuous States

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Abstract: Hybrid mathematical models are often represented as continuous functions with discontinuous inputs, or they are visualised as state machines or petri-nets comprising continuous models linked by discontinuous mappings. The analysis and simulation of hybrid (or nonsmooth dynamical) models is plagued with difficulty, necessitating careful consideration of energy losses and state reinitialisation on commutation. The author proposes an alternative model, where states are discontinuous. The engineer familiar with techniques such as signal flow graphs or bond graphs can clearly visualise discontinuities as breaks (or joins) in power flow between parts of the model. A mixed-Boolean state equation can be derived which reflects the physics of switching behaviour. This has two advantages: first, by considering the physics incrementally about the discontinuity it can be simulated without the need for state reinitialisation algorithms, and second, it can be analysed for structural control properties to show how they change with commutation.

Keywords: Hybrid models, State space methods, Switching variables, Systems models, Bond graphs.

1. INTRODUCTION

Hybrid models are those containing continuous and discontinuous functions. They are used to model variable structure systems (such as contact), and those where rapidly changing nonlinear behaviour can be described by some form of piecewise continuous equation (such as stiction/friction).

Hybrid systems can be visualised as continuous modes on areas of state space linked by a discontinuous state mapping (Mosterman et al., 1998) and described as a hybrid automaton i.e. one that contains both finite and continuous state spaces (Van Der Schaft and Schumacher, 1999). The dynamics consist of discrete transitions plus an evolution of the continuous part in each location.

There are many ways of abstracting a physical system to a hybrid model and, it appears that the diversity of methods reflect the many applications and tools available.

Sliding Mode Control for Variable Structure Systems assumes systems "governed by ordinary differential equations with discontinuous state functions in the right hand side" (i.e. the input) (Utkin, 1992). It is well established that a discontinuous control action typically in the form of a switching input causes the systems structure to vary. A subspace or hyperplane (the switching surface) divides the state space of the model into two regions, each with a different control law (or form of). When the system operates on the switching surface, it is said to be in sliding mode and sliding control utilises this idea to give robust control in discontinuous and nonlinear systems. This method can be extended to variable-structure systems where the parameters - and not just the control inputs - are discontinuous (Marghitu and Irwin, 2001).

Petri-nets can be used to describe a set of interlinked continuous models, but these can become large: 2^n models where n is the number of switches (Borutzky, 1995).

The linear complementarity problem, comprising a continuous equation (such as a state equation) and complementarity condition (Van Der Schaft and Schumacher, 1999) is perhaps the most widely used model in the field of nonsmooth dynamics. These contain an external signal which can be thought of as a Lagrange multiplier, and commutate between zero and a value which must be calculated. This model can be transferred to a single inclusion or a variational inequality, which have unique continuous solutions.

Mixed logical dynamical (MLD) systems i.e. those with interdependent physical laws, logic rules, and operating constraints, have been established (Bemporad and Morari, 1999) and shown to be equivalent to other classes of hybrid system (Heemels et al., 2001). The model presented here differs in that it originates from idealised physical modelling (i.e. the Bond Graph) and explicitly embraces the physics and changing causality of the system.

Users of commercial-off-the-shelf software can naively use 'switches' in a model without appreciating the impact this has. In the author's experience this frequently happens, with an inexperienced engineer or researcher left attempting to tackle algebraic loops or integration errors by randomly inserting transfer functions and sources of compliance before a pressing deadline. Clearly, this is an unacceptable strategy. The author's work on Bond Graph methodology was motivated by the need to promote deep understanding of both physics and computational considerations among analysts, and informs this research.

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Willems presents a case for using idealised physical modelling methods (of which bond graphs are an example) to mitigate against unwittingly creating physically meaningless or computationally inefficient models (Willems, 2007).

The author argues that, in cases where the discontinuity is an integral part of the physical system (a switch, mechanical contact, or some highly nonlinear behaviour abstracted by the modeller to a heaviside function), it is more physically representative for the state variables to be discontinuous.

2. CATEGORISATION OF DISCONTINUITIES

Branicky et al (Branicky et al., 1998) categorise hybrid models into *Switching* and *Impulse* models, which can be *controlled* or *autonomous*.

Switching models are defined as those where the vector field changes discontinuously when the state hits a boundary. Switching systems "comprise a family of dynamical subsystems together with a switching signal determining the active system at a current time" (Vu and Liberzon, 2008). They are a subset of hybrid systems, where there is some discontinuous behaviour modelled by an on/off switch or other binary signal.

Impulse models are those where the continuous state changes impulsively on hitting prescribed regions of state space. The classic example is Newtons Collision law, where the state of a body changes from positive to negative velocity on impact, and any dissipative effects are accounted for by a coefficient of restitution. The state changes impulsively, and there is an impulse loss on commutation.

The author proposes a further distinction between *Structural* and *Parametric* discontinuities (Margetts, 2013).

Proposition 1. Structural Discontinuities occur when parts of the model are connected or disconnected, interrupting power flow between components. These discontinuities often give rise to variable topology models.

Engineering examples of structural discontinuities are the hydraulic valve, mechanical clutch, ideal electrical switch, or contact between bodies.

Proposition 2. Parametric Discontinuities occur when an element has a highly nonlinear constitutive equation, which has been abstracted to a piecewise continuous function. The structure of the model is unchanged, it is the equation describing the behaviour of an element which changes.

Common examples of parametric discontinuities are dry friction, tyre forces, a two–stage oleo strut 'breaking out,' or saturation of an electrical capacitor or hydraulic accumulator.

3. DERIVING THE MIXED-BOOLEAN MODEL

Bond Graphs were instrumental in deriving the mixed-Boolean model. Bond Graphs are an idealised physical modelling method, enabling the user to sketch a system, assign computational causality, and derive a state space model. Readers unfamiliar with the technique are directed to Karnopp, Margolis and Rosenberg's standard text (Karnopp et al., 2006).

There has been a significant body of work on Hybrid Bond Graphs, with numerous variations proposed and extensive discussion of their implications for structural analysis and simulation. The author's work grew from a desire to construct hybrid models which accurately reflect the physics of the system – offering the user insight – as well as being suitable for accurate simulation. Previously, none of the proposed Hybrid Bond Graphs achieved this. A thorough literature review is given in (Margetts et al., 2013).

A form of Hybrid Bond Graph can be defined which incorporates 'controlled junctions' for structural discontinuities (Margetts et al., 2013) and 'controlled elements' (containing a mode–switching 'tree' of junctions and elements) for parametric discontinuities (Margetts and Ngwompo, 2015). The control signals are assigned a Boolean parameter, which sets each junction to an ON or OFF state (1 or 0). When ON, power flows through the junction uninhibited. When OFF, the junction is effectively replaced with null sources or sinks.

Equations relating the elements in the system can hence be written in terms of Boolean parameters. The equations fully describe the system in all potential modes of operation: when Boolean parameters are 'ON' or 'true,' the equations are multiplied by one, but when they are 'OFF' or 'false' the equations are multiplied by zero and cease to be part of the model for that mode of operation.

One important point highlighted by use of the hybrid bond graph is the presence of *dynamic causality*. Bond graphs are an acausal modelling method i.e. the model is constructed *before* inputs and outputs are defined. This means that the state space model derived from the bond graph can be put in 'preferred integral causality' so as to aid computation. However, in hybrid models the ideal 'preferred integral causality' assignment can change on commutation. Many users seek to constrain dynamic causality, usually by adding sources of compliance or 'causality resistance' (Asher, 1993; Breedveld, 2000, 2002). This is somewhat controversial as it can yield stiff models and slow simulation times, while disregarding important physical information (Cellier et al., 1994; Buisson, 1993). For example, in the case of rigid contact the dynamic causality reflects the genuine kinematic constraint between two bodies (which can temporarily be considered as a single rigid body).

For the Hybrid Bond Graph. The same rules for deriving a state space model from a regular bond graph are followed. This derivation revolves around turning the graphical bond graph model into a 'Junction Structure Matrix.' This is a matrix of 1's and 0's which link all of the inputs and outputs in the model. The difference with the Hybrid bond graph is that a Boolean term can be inserted. For parametric discontinuities, Boolean terms are contained in the expression for an element. For structural discontinuities, Boolean terms are introduced to the Junction Structure Matrix where the relationship between inputs and outputs depends on commutation. The matrix equation equation is then rearranged into state space form, by placing it in terms of the states (which are the inputs to the storage elements). A generalised Junction Structure Matrix is shown in equation (1). Fig. 1 shows the key