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Research paper

Stochastic resonance in dissipative drift motion

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ABSTRACT

We study a simple model of drift waves that describes the particle transport in magnetised plasmas. In particular, we focus our attention on the effects of noise on a dissipative drift wave model. In the noiseless case, the relationship between the escape time and the damping term obeys a power-law scaling. In this work, we show that peaks in the escape time are enhanced for certain values of the noise intensity, when noise is added in the dissipative drift motion. This enhancement occurs in the situation where stochastic resonance (SR) appears. We also observe that the noise produces significant alterations to the escape time distribution. This way, we expect this work to be useful for a better understanding of drift wave models in the presence of noise, since noise is a natural ingredient in the environment of this kind of physical problems.

1. Introduction

Drift waves can appear in magnetically confined plasmas [1]. They have a frequency lower than the frequencies related to magnetohydrodynamic waves, and they have been widely investigated in anomalous diffusion across magnetic fields [2], mainly in drift wave turbulence in toroidal plasmas [3,4]. An experiment in a plasma where collisional drift waves were identified, has been carried out in [5]. Besides, a relationship between collisional drift waves and enhanced plasma transport was observed. A theoretical description of drift waves has been proposed by Horton [6], where conditions on the drift wave for stochasticity and the validity of the diffusion approximation have been proposed.

Drift waves for a large aspect ratio tokamak can be described by two-wave Hamiltonian model. In the present work, we have included modifications in the two-wave Hamiltonian model, which we describe now. The two-wave Hamiltonian model has been considered to describe particle transport [2]. This model can exhibit island chains in phase space, as well as chaotic behaviour [7]. The effect of dissipation was previously studied in Ref. [8]. In this work, we modify the model including not only a dissipative term, but also a noisy term aiming at studying the escape times in this system. In spite of the importance of the effect of noise in the dynamics, this has been scarcely considered in previous works on the two-wave Hamiltonian model.

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Certainly, the effects of noise on dynamical systems are relevant in science and engineering [9]. Precisely, one of the effects of noise is the well-known phenomenon of stochastic resonance (SR). It mainly consists in the appearance of a cooperative effect between the internal mechanism and the external forcing [10,11], and as a consequence SR improves a system performance measure. SR has been observed and studied in many systems, leading to important experimental and theoretical results [12]. Park and Lai [13] showed that a common feature of SR is the high sensitivity to the variation of noise. Gluckman and collaborators [14] experimentally demonstrated the existence of SR in neuronal networks from the mammalian brain. The SR has been also studied in a large variety of systems, such as climatic change [15], biological cells [16], and more recently in problems of periodic potentials [17] and anharmonic oscillators with coexisting attractors [18].

The purpose of this article is to investigate the effects of noise on the escape time in the dissipative drift motion. In our context, noise can represent the collision effect between particles [19]. The escape time features in Hamiltonian drift waves were studied in Refs. [20–23]. Escape time in a dissipative drift wave model without noise was already analysed by Oyarzabal et al. [8], where a power-law in the escape time as a function of the dissipation parameter was found. The main conclusions of this work are the presence of SR and the extreme sensitivity of the escape time, as a function of the noise intensity. Therefore, the escape times are large in this SR situation, and as a result particles are trapped for a long time.

The paper is organised as follows. In Section 2, we briefly describe the drift wave model and we introduce a noise term in the dissipative model. In Section 3, we verify some SR characteristics in the dissipative drift motion with noise, where we observe that the escape time can be maximal when the noise intensity reaches a finite level. Finally, the conclusions are presented in Section 4.

2. Effect of noise in the dissipative drift motion

Here we analyse a model of drift waves that describes the transport of particles in a magnetically confined plasma. In a magnetised plasma the electrostatic wave propagation is in the poloidal direction and the drift velocity of the guiding centers is given by [6]

$$\vec{v} = \frac{\vec{E} \times \vec{B}}{B^2}, \quad (1)$$

with electric field $\vec{E} = -\nabla\phi$, constant toroidal magnetic field $\vec{B} = B_0\hat{e}_z$, and electrostatic potential

$$\phi(x, y, t) = \phi_0(x) + \sum_{i=1}^{N} A_i \sin(k_{xi}x)\cos(k_{yi}y - \omega_i t + \beta_i), \quad (2)$$

where A_i is the amplitude of the ith wave, k_{xi} and k_{yi} are the wavenumbers in the x direction (radial coordinate) and y direction (poloidal coordinate), respectively. Furthermore, ω_i is the angular frequency, β_i is the phase, N is the number of electrostatic drift waves propagating in the poloidal direction y, and $\phi_0(x)$ is the equilibrium electrostatic potential that depends on the radial electric field profile. In our case, we have used a uniform electric field $\phi_0(x) = ax$. Eq. (2) is a simple nonlinear description based on computer simulations and analytical studies reported by Horton in drift waves [24]. With regard to phases, Bénisti and Escande [25] considered initial random phases in a set of electrostatic waves to study diffusion properties of the standard map, for instance the case in the beam-plasma instability. The influence of the phases on the motion of a particle in the field of many waves was investigated by Elskens [26]. In our simulations, we consider the phases with values equal to zero according to work about drift waves and transport by Horton [2]. Batista et al. [27] also used phases equal to zero in a nonlinear three-mode interaction model to analyse the occurrence of drift wave turbulence driven by pressure gradient in the edge plasma of a tokamak [28]. In our work, for $\beta_i = 0$, the model exhibits a rich dynamical behaviour with chaotic and periodic dynamics, which can be used to describe drift wave driven transport.

Substituting the electrostatic potential into the Hamiltonian $H(x, y, t) = \phi/B_0$ and introducing the variables

$$x' = \frac{x}{r_0}, y' = \frac{y}{r_0}, a' = \frac{a}{E_0}, t' = \frac{t}{t_0},$$
$$\omega' = \omega_i t_0, A'_i = \frac{A_i}{E_0 r_0}, \vec{k}'_i = \vec{k}_i r_0, \quad (3)$$

we obtain the dimensionless Hamiltonian [6]

$$H(x, y, t) = ax + \sum_{i=1}^{N} A_i \sin(k_{xi}x)\cos(k_{yi}y - \omega_i t). \quad (4)$$

This system is integrable and exhibits stable orbits when $N = 1$. For $N > 1$ the system is integrable if the phase velocities are the same, and it is not integrable if at least one phase velocity is different. In the nonintegrable case the system exhibits chaotic dynamics.

We find the equations of motion for the drift caused by two waves from the dimensionless Hamiltonian. In these equations, we have introduced dissipation as in Ref. [8] and a noise term. Dissipation is the result of many processes that can be due to ion viscosity, ion-ion or ion-neutral collisions, ion Landau-damping, and coupling to damped modes such as ion-acoustic waves [29]. Kabantsev et al. [30] verified that chaotic collisional neoclassical transport causes enhanced dissipation