Delay induced steady-state transition and stochastic resonance for an ecological vegetation growth system subjected to multiplicative and additive noises[☆]

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ABSTRACT

In this paper, our aim is to investigate the steady state properties and stochastic resonance (SR) phenomenon for an ecological vegetation growth system with time delay induced by the multiplicative and additive noises. Numerical results show that the SR phenomenon caused by time delay, different noise terms and a weak periodic signal occurs in the vegetation growth model under different values of system parameters. With regard to the stationary state properties of the vegetation system, the results indicate that the terms of different noises and time delay can all accelerate the shift from the substantial state to the barren one of the ecological system, restrain the development of the vegetation system and weaken the stability of the ecological system. On the other hand, the additive noise strength always enhances the SNR and the SR phenomenon, while the intensity of multiplicative noise often reduces the effect of the SR. In particular, time delay can play different roles in exciting the SR phenomenon in different cases.

1. Introduction

The effect of SR was initially discovered in the research of the periodic changes of the ancient climate [1]. Afterward, SR phenomenon has been extensively and systematically discussed [2–4]. In fact, the SR phenomenon has appeared in a great number of non-linear physical and biological systems with the extrinsic and intrinsic perturbations [5–7]. For instance, SR phenomenon frequently occurs in climate transitions [8], system size of general nonequilibrium potential framework [9], coupled or extended systems [10–13], monostable nonlinear systems [14,15], the case of non-Gaussian noise [16] . It was concluded that non-linearity, periodic and random force are the essential ingredients for the onset of SR. On the other hand, the behavior similar to SR has been found in linear systems subjected to multiplicative colored or dichotomous noise [17–19]. The classical behaviors of SR lie in the fact that the one or two maxima of the signal-to-noise ratio (SNR) as a function of the terms of the noise intensity or the time delay often exist in the nonlinear systems.

It is worth noting that the investigations of SR phenomenon has been recently introduced into the nonlinear systems with time-delay induced by the multiplicative and additive noises, which revealed an important fact that the time delay can produce significant

influences on the SR effect [20–23]. In fact, a variety of stochastic systems include time-delayed feedback. In the light of such feedback loops, signal or other key quantities of the systems are fed back from output interfaces. The SR effect in time-delay systems has been investigated by many of scholars [24–31]. The meaning of their research lies in the fact that it is more reasonable that time delay is included in some actual systems for the transfer of information, matter, energy of the nonlinear systems from output to input interfaces needs a certain time in the natural world. The way of approximation analysis for time-delayed stochastic systems has been extended to the case of small delay time [32].When the delay time is large, a numerical simulation method can be applied [33]. For another, it should be noted that in many previous investigations on SR the signal was often introduced into the system in an additive way. In fact, SR excited by the multiplicative signal has started to be discussed in some systems without time delay [34–37], which implies that the intervention of a multiplicative signal can generate some new behaviors of SR. Recently, Zeng et al. [38–42] have investigated the impact of intrinsic and extrinsic noises on the SR and the properties of stationary states in the time-delayed vegetation model and the tumor cells which shows that noises and time delay could induce resonance phenomenon in an ecological system with a weak multiplicative periodic signal. Jia and Mei [43] have also studied a delay-induced catastrophic regime shifts in ecosystems. Besides, Wang and Matjaž et al have also investigated the effects of noise and time delay on the SR and the synchronization extensively for neuronal networks [46–50]. Recently, the effects of different types of noises on the SR for the neural network systems and other biological nonlinear systems have also been investigated in Refs [51–59].

In the present paper, on the basis of the stochastic vegetation development model, the shift between the substantial state and the barren one, the SR phenomenon with consideration of the additive noise, multiplicative noise, the time delay and a multiplicative periodic signal are discussed. The effects of the noises and time delay on SR induced by the additive signal are studied in terms of the SNR technique. The paper aims to contribute to a clear understanding of the dynamical behaviors of the vegetation under the influences of the additive and multiplicative noises, time delay and a periodic signal. In the second section, we introduce the stochastic vegetation growth system including the noises and time delay. In the third section, the steady state properties for the vegetation growth system in the presence of Gaussian white noises and time delay are discussed. In the fourth section, the approximate analytical expression of the SNR of the system is derived, and the impacts of all kinds of the noises and time delay on the SNR of the stochastic system are analyzed. A brief conclusion and some discussions are depicted in the final section.

2. Stochastic ecological vegetation system including noises and time delay

In accordance with the Shnerb's [44] model of vegetation dynamics, [45], we can give the determinative dynamical equation of the ecological vegetation model as follows:

$$\frac{dB}{dt} = \rho B \left(\frac{R}{1 + \alpha B} - \frac{B}{B_c} \right) - \mu \frac{B}{B_0 + B}, \tag{1}$$

where B denotes the biomass of vegetation, R stands for the mean annual rainfall. ρ represents the logistical growth rate for the vegetation biomass. α is called the water consumption rate by the biomass, B_c is the carrying capacity of the biomass, μ is denoted by the grazing loss rate, and B_0 is defined by a biomass, at which the loss rate is half maximum. In fact, the grazing loss rate is always influenced by all kinds of external environmental factors. Therefore, we can rewrite the coefficient μ as $\mu + \xi(t)$, where $\xi(t)$ signifies the fluctuation of the external climate, the temperature, the sunshine, the forest fire and so on. $A\cos(\omega t)$ represents the external periodic force from the nature. $\eta(t)$ is on behalf of the interaction between the vegetation biomass competing for water, nutrient and sunshine. In the meantime, taking into consideration the factor that water and the nutrient need some time to diffuse in the vegetation biomass, it is sound to introduce a time delay term τ into the vegetation growth model. Therefore, the stochastic differential equation for the cancer development can be described as:

$$\frac{dB}{dt} = \frac{\rho B R}{1 + \alpha B} - \frac{\rho B_\tau^2}{B_c} - \mu \frac{B}{B_0 + B} - \xi(t) \frac{B}{B_0 + B} + \eta(t) + A\cos(\omega t), \tag{2}$$

where B_τ denotes the time delayed variable with $B_\tau = B(t - \tau)$. $\xi(t)$ and $\eta(t)$ are the Gaussian white noise and characterized by their mean and variance, i.e

$$
\begin{aligned}
\langle \xi(t) \rangle &= \langle \eta(t) \rangle = 0, \\
\langle \xi(t)\xi(t') \rangle &= 2Q\delta(t - t'), \\
\langle \eta(t)\eta(t') \rangle &= 2M\delta(t - t'),
\end{aligned}
\tag{3}
$$

where both of Q and M are the intensities of the multiplicative noise and the additive noise respectively. $A\cos(\omega t)$ is a weak low-frequency signal, i.e. $A \ll 1$ and $\omega \ll 1$. A is the amplitude of the input periodic signal and ω is its frequency. Eq. (1) can be considered as describing an overdamped motion of the state variable moving in a quasi-'free energy potential':

$$V(B) = \frac{\rho B^3}{3B_c} - \left(\frac{\rho R}{\alpha} - \mu \right) B + \frac{\rho R}{\alpha^2} \ln(1 + \alpha B) - \mu B_0 \ln(B_0 + B), \tag{4}$$

When the perturbations of noise terms and time delay term are ignored, the fixed points of Eq. (1) which dependent wholly on the above parameters are $B_{s1} = 0$ (barren state), and $B_{s2} \approx 5.88633$ (sustainable vegetation state), another unstable point is $B_u \approx 0.45168$.