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# Performance investigation of stochastic resonance in bistable systems with time-delayed feedback and three types of asymmetries

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## HIGHLIGHTS

- SR in a bistable system with three types of potential asymmetries and time-delayed feedback is studied.
- Output SNR of modified bistable system is derived.
- SR effect and SNR peak depend on asymmetric types and time-delayed feedback.
- Well-width asymmetry can control SR more effective than well-depth asymmetry.

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## ABSTRACT

The simultaneous influence of potential asymmetries and time-delayed feedback on stochastic resonance (SR) subject to both periodic force and additive Gaussian white noise is investigated by using two-state theory and small-delay approximation, where three types of asymmetries include well-depth, well-width, and both well-depth and well-width asymmetries, respectively. The asymmetric types and time-delayed feedback determine the behaviors of SR, especially output signal-to-noise ratio (SNR) peaks, optimal additive noise intensity and feedback intensity. Moreover, the largest SNR in asymmetric SR is found to be relatively larger than symmetric one in some cases, whereas in other cases the symmetric SR is superior to asymmetric one, which is of dependence on time delay and feedback intensity. In addition, the SR with well-width asymmetry can suppress stronger noise than that with well-depth asymmetry under the action of same time delay, which is beneficial to weak signal detection.



## 1. Introduction

The employment of noise in enhancing weak signal detection has attracted sustaining attention [1–4] in various fields such as biology [5,6], medicine [7], optics [8], communications [9,10] and mechanics [11–13]. Traditionally, the methods based on noise cancellation attempt to suppress or cancel the noise and further highlight weak signals masked in noise. Since they inevitably endamage weak signals more or less in the de-noising process, the performance on weak signal detection

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is unsatisfactory under strong background noise. Unlike traditional signal processing techniques, stochastic resonance (SR) [14–18], as a potential tool for signal processing [19], is able to utilize the noise to enhance and detect weak signals overwhelmed by strong background noise [20]. Since SR possesses the unique behavior that weak input signals are able to be amplified and optimized by the addition of moderate noise, it has attracted considerable interest, especially in theoretical research [21–24] and experimental investigation [25–27].

In the course of an ever-increasing flourishing of SR, new theoretical studies and applications with novel types of SR have been discovered, and there seems to be no end in sight. On the one hand, since the asymmetry in real physical or natural systems is widespread, the research on asymmetric systems has aroused much popular interest. Originally, Wio and Bouzat [28] deduced the traditional two-state theory into asymmetric bistable systems, thereby using mean first passage time (MFPT) to evaluate the influence of asymmetries on SR. Afterwards, They [29] indicated that potential symmetry can improve the output signal-to-noise ratio (SNR) of a three-field reaction–diffusion system driven by additive Gaussian white noise. Zhang and Xu [30] studied the SR of an asymmetric bistable system subject to periodic rectangular signal and colored noises. Moreover, Li et al. [31] analyzed the influence of asymmetry on the noise enhanced stability, where an asymmetry parameter is introduced into a symmetric bistable system. Borromeo and Marchesoni [32] explored the SR in a deformable asymmetric double-well system. In addition, Guo et al. researched the SR effect in a piecewise nonlinear model [33] and a second-order underdamped asymmetric bistable model [34], respectively. The above mentioned research on asymmetric SR just considers the effect of synchronous asymmetries of both well depth and width on SR, but the individual difference between well-depth and well-width asymmetries in SR has not been explored. Recently, Qiao et al. [35] studied and discussed the effect of three types of asymmetries on increasing SNR respectively, suggesting that some asymmetries can amplify and enhance the resonant peaks of SR, which is helpful to detect weak signals. On the other hand, however, stochastic systems with time-delayed feedback also have attracted much attention in various fields, such as stochastic resonance with delayed interactions [36], synchronized and coordinated movements with time delay [37], laser systems with optical feedback [38], feedback-regulated voltage controlled oscillations [39], and etc. In these systems, the time delay mainly arises due to a finite transmission speed of matter, energy, information and so on. For example, Wang et al. [40] studied the effect of time-delayed feedback on the performance of SR in weak signal detection. Moreover, Guilouzic et al. [41] deduced the time-delayed Fokker–Planck equation (FPE) with small time delay and Frank [42] improved the expression on the basis of perturbation theory. Shao and Chen [43] investigated the SR in a bistable system with time-delayed feedback driven by a weak periodic force, suggesting that the time delay could enhance SR phenomenon, whereas Guo et al. [44] stated that static asymmetry could restrain the occurrence of SR in an asymmetric bistable system subject to mixed periodic signal and time-delayed feedback. In addition, Wu and Zhu [45] investigated the phenomenon of SR in a bistable system with time-delayed feedback, where the number of peaks in quasi-steady-state probability distribution depends on both external driving forces and time delay.

Though there are many studies on SR in either asymmetric or time-delayed bistable systems, the performance of bistable SR under the synchronous action of both potential asymmetries and time-delayed feedback have not been explored, especially the difference between well depth and well width. Therefore, the present letter attempts to study the influence of certain forms of potential asymmetries on bistable SR with time-delayed feedback. For this purpose, the bistable potentials with three different types of asymmetries are initially introduced in Section 2, which include well-depth asymmetry, well-width one, and both well-depth and well-width one, respectively. Then, Section 3 derives the analytic expression of output SNR in the SR under the simultaneous action of both three types of asymmetries and time-delayed feedback by virtue of two-state theory and small-delay approximation. In addition, Section 4 evaluates and analyzes the role of time-delayed feedback and asymmetries in SR subject to additive Gaussian white noise and periodic force. Finally, conclusions are drawn in Section 5.

## 2. The bistable potentials with three types of asymmetries

Traditional asymmetric SR has been studied extensively by tilting an otherwise symmetric double-well potential and the widely used potential function is [34–36]

$$U(x) = -\frac{a}{2}x^2 + \frac{b}{4}x^4 + \alpha x \tag{1}$$

where $a$ and $b$ are system parameters, and $\alpha$ is an asymmetric factor where $a, b, \alpha > 0$. Fig. 1(a) shows traditional bistable potentials with an asymmetric term under different $\alpha$ where $a = 1$ and $b = 0.5$. One can easily observe that the depth and width of double wells vary simultaneously as asymmetric factor $\alpha$ increases. Such a behavior suggests that traditional asymmetry only considers the synchronous effect of both well-depth and well-width asymmetries on SR, but it still no investigates the influence of well-depth and well-width asymmetries on output SNR alone. To investigate the difference among well depth, well width and their combination in a bistable system subject to both time-delayed feedback and asymmetries, a bistable potential function with three different types of asymmetries is introduced as below [35]

$$U_{oi}(x) = \begin{cases} -ax^2/2 + bx^4/4, x \geq 0 \\ -aA_i x^2/2 + bB_i x^4/4, x < 0 \end{cases} \tag{2}$$