Chinese Journal of Physics 000 (2016) 1–9

Contents lists available at ScienceDirect

Chinese Journal of Physics

journal homepage: www.elsevier.com/locate/cjph

Dynamical complexity and stochastic resonance in an asymmetry bistable system with time delay

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ARTICLE INFO

Article history:
Received 17 September 2016
Revised 19 October 2016
Accepted 19 October 2016
Available online xxx

Keywords:
SR
Asymmetric bistable system
Dynamical complexity
Time-delayed

ABSTRACT

The dynamical complexity and stochastic resonance (SR) of a time-delayed asymmetric bistable system are studied. Firstly, The effective potential function and steady-state probability density function are deduced based on Born-Oppenheimer approximation theory, and we find that the asymmetric item and time-delayed feedback item can both affect the curve of these two functions, especially the asymmetric item can induce phase displacement. Secondly, the mean first-passage time (MFPT) which plays an important role in research on particles escape rate is derived and we obtain an approximate asymmetric item r which can maintain a steady MFPT. Finally, the influences of different parameters on SR are researched by signal-to-noise ratio (SNR). The analytic expression of SNR is derived and three dimensional graphs and contour maps of SNR with different parameters are obtained. The results indicate that time delay τ and time delay strength e can enhance the SNR and the asymmetric item r has a non-monotone effect on SNR. Notably, adjusting time delay strength e is more sensitive than that of the time delay τ in controlling SR.

1. Introduction

Stochastic resonance (SR) was originally studied in the early 1980s by Benzi and Nicolis for explaining the rules of ice ages [1–3]. Subsequently, plenty of experiments and theory researches on SR were springing up [4–9]. Up to now SR has been applied to various fields [10–19]. He et al. [10] investigated SR in the over-damped fractional oscillator subject to multiplicative dichotomous noise. Silchenko et al. [12] used a multifractal formalism to study the effect of stochastic resonance in a noisy bistable system driven by various input signals and led us to consider the degree of multifractality as a new measure of stochastic synchronization. Bag et al. [13] discussed the mean lifetime for the escape of a Brownian particle through an unstable limit cycle driven by multiplicative colored Gaussian and additive Gaussian white noises.

SR is a phenomenon which can enhance the output signal-to-noise ratio (SNR) when an optimal level of noise is introduced into a nonlinear dynamical system with a weak external signal. Traditional theories on SR were most about ideal systems which ignored the influence of time delay factor. Time delay is ubiquitous in physical systems, chemical systems, biological systems, nervous systems, mechanical systems, engineering control systems and so on. Recently, the stochastic system with time delayed feedback has been a hotspot for its potential applications [20–32]. Wang et al. [20,21] introduced

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http://dx.doi.org/10.1016/j.cjph.2016.10.013
0577-9073/

time delay into SR and concluded that time delay could produce different effects on SR under the conditions of different parameters. Guilouzic et al. [26] deduced the time-delayed Fokker–Planck equation with small delay and Frank [27] improved the expression on the basis of perturbation theory. Chen et al. [28] theoretically studied a bistable system with time-delayed feedback driven by a weak periodic force concluding that the time delay could enhance SR phenomenon. He et al. [29] utilized statistical complexity measure and normalized Shannon entropy to explain SR phenomenon and found an effective method for quantifying SR in time-delayed bistable systems. Guo et al. [30] studied the effects of time delay in a bistable system driven by square-wave signal and got a critical value of time delay which had a positive or negative effect on SR. Zhang et al. [31] investigated the SR in a delayed asymmetry bistable system by two-state theory. Guo et al. [32] stated that static asymmetry r could restrain SR, and the time delay could enhance SR in delayed asymmetry bistable systems.

Most systems in nature are not symmetrical. SR in asymmetrical systems has attracted an increasing attention in recent years [33–38]. Yang et al. [34] investigated SR in an asymmetric bistable system driven by colored noise and found that the SR phenomena appeared in most cases and disappeared in some special cases. Long et al. [37] studied SR in an asymmetric bistable system, and found that SR phenomenon could be weakened by the asymmetry item. Yao et al. [38] researched on SR in a bias mono-stable system driven by a periodic rectangular signal and uncorrelated noises, and found that SR phenomenon was more evident with increasing of $|r|$.

This paper studies the dynamical complexity and SR in an asymmetry bistable system with time delay. The rest of this paper is organized as follows: in Section 2 we derive the potential function and stationary probability function of time-delayed asymmetry bistable system, and discuss the influence of parameters. In Section 3 we deduce the mean first-passage time (MFPT) of the system, and obtain an approximate value of r which can maintain MFPT unchanged. Then in Section 4 the function of SNR is derived and three dimensional figures and their contour maps are described to study the affection with parameters especially the asymmetric item r and the time delay τ. Finally we draw conclusions in Section 5.

2. Time-delayed asymmetry bistable system and its complexity

A classical symmetry SR system is shown as

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\mathrm{d}U(x)}{\mathrm{d}x} + S(t) + N(t) \tag{1}$$

Where $N(t) = \sqrt{2D}\xi(t)$ is Gaussian white noise with $< \mathrm{N(t)N(t+\tau)} > = \sqrt{2D}\delta(t)$, in which D represents the noise intensity with zero mean and unit variance. $S(t)=A\cos(\omega t+\phi)$ is periodic signal, in which A represents the amplitude, and ω denotes the driving frequency, and ϕ is the phase.

The typical $U(x)$ is a reflection-symmetric potential written as below

$$U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4 \tag{2}$$

where a and b are the two items which influence the bistable potential barrier and potential well.

When the asymmetric item and a time-delayed feedback item are introduced into Eq. (2), the time-delayed asymmetric SR model can be governed by [39]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = ax(t) - bx(t)^3 + r + ex(t-\tau) + A\cos(\omega t + \varphi) + \sqrt{2D}\xi(t) \tag{3}$$

So a time-delayed asymmetric bistable system can be expressed as

$$U(x) = -\int^a x(t) - bx(t)^3 + r + ex(t-\tau)dx = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4 - rx - ex(t-\tau)^2 \tag{4}$$

Without loss of generality, we set $a=1$, $b=1$.The Fokker–Planck equation is

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial[A(x)]P(x,t)}{\partial x} + \frac{\partial^2[B(x)P(x,t)]}{\partial x^2} \tag{5}$$

where

$$A(x) = \int_{-\infty}^{+\infty} \mathrm{d}x_\tau (x - x^3 + r + ex_\tau + A\cos(\omega t))P(x_\tau, t-\tau|x,t) \tag{6}$$

$$B(x) = D \tag{7}$$

and in Eq. (6) $x_\tau = x(t-\tau)$, $P(x_\tau, t-\tau|x, t)$ is the zero order approximate Markovian transition probability density that can be described as the following function [28,40]

$$P(x_\tau, t-\tau|x,t) = \frac{1}{\sqrt{4\pi D\tau}} \exp\left(-\frac{(x_\tau - x - A(x)\tau)^2}{4D\tau}\right) \tag{8}$$

where

$$C(x) = x - x^3 + r + ex \tag{9}$$