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Forced oscillations of cracked beam under the stochastic cyclic loading

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ABSTRACT

An analysis of forced oscillations of cracked beam using statistical methods for periodically correlated random processes is presented. The oscillation realizations are obtained on the basis of numerical solutions of differential equations of the second order, for the case when applied force is described by a sum of harmonic and stationary random process. It is established that due to crack appearance forced oscillations acquire properties of second-order periodical non-stationarity. It is shown that in a super-resonance regime covariance and spectral characteristics, which describe non-stationary structure of forced oscillations, are more sensitive to crack growth than the characteristics of the oscillation's deterministic part. Using diagnostic indicators formed on their basis allows the detection of small cracks.

1. Introduction

Investigation of oscillations of damaged elastic bodies allows one to analyze changes in the vibration state of construction elements during operation and so methods for such damage detection can be developed on this basis. In such investigations, special attention is paid to defining of oscillation parameters in the case of fatigue cracks that close and open during the deformation [1–14]. This leads to changes in the elastic and inertial characteristics of the construction elements and is a reason of oscillation's nonlinearity. The beam with asymmetrical breathing crack is strongly nonlinear system with bilinear stiffness. The nonlinear feature of forcing oscillations of cracked beam under harmonic excitation was analyzed analytically and experimentally in [1–4,9–16]. The detail numerical study using two-dimensional finite elements model (2-D FEM) has been performed in [14]. It was shown that dynamic response is very rich of sub- and super-harmonic components. In the case when forcing frequency is close to anyone of integer multiples $(1/n)$ of the first of the bilinear frequency ω_b, then the n-th harmonic component of the forced oscillation, which is close to ω_b, will be significantly exalted [9,11–19]. The appearance of the sub- and super-harmonic components related to the occurrence of damage was experimentally tested in FFT spectra of forced vibration too [14]. Thus using of sub- and super-harmonic components allows to improve the crack detection technique. The damage detection procedure was summarized in [14] as follows: (1) the estimation of bilinear frequency of the cracked beam; (2) the excitation of forced vibration with super- and sub-resonance frequencies; (3) evaluation

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of the nonlinear damage indicators (NDIs), e.g. the ratio between the magnitude of super- or sub-harmonic component and the fundamental harmonic component.

In the mentioned above paper the analysis of nonlinear effects, which occur only under harmonic excitation of the cracked beam, is provided. The aim of the present paper is to study properties of oscillation excited by a loading in the form of the sum of harmonic and stochastic parts. The stochastic part is described by a narrow-band stationary random process. The chosen model of excitation is one of the simplest individual cases of the periodically correlated random processes (PCRP), methods of which are widely used for the analysis of the rotary mechanisms vibration with a purpose of detection of their faults on the early stage of initiation [31–41]. These processes are also called cyclostationary [31–41]. They are periodically nonstationary only with respect to mean function. For such cyclic stochastic excitation it is expected that nonlinearity of the system will lead not only to the appearance of new harmonic components in deterministic part of a signal but also to changing of the stochastic part and to their interaction. It is assumed that such interaction appears in stochastic modulation of harmonics of deterministic part. Such modulation can be detected and quantitatively described using methods of statistical analysis of PCRP [17–30]. The results obtained using such approach are the subject of analysis of the present paper. The investigation of the changes, which occur in harmonic composition of the deterministic part, that is described by a mean function of PCRP, as well as in the covariance-spectral structure of the stochastic part for the case, when crack appears and grows is carried out. Such structure can be described by the Fourier coefficients of the covariance function and spectral density – by so-called covariance and spectral components [28,29]. These quantities quantitatively characterize periodic nonstationarity of the second order. Their values are equal to zero in the case of uncracked beam. Comparative analysis of covariance components sensitivities and the amplitudes of harmonics of deterministic part showed the advantage of the former. It allowed to propose new more effective indicators for crack detection, which are formed on the base of characteristics of the periodic nonstationarity of the second order. Using of linear band filtration [25,26] the separation of the stationary random processes, which stochastically modulate harmonics, is provided in the paper and the estimators of their auto- and cross-covariance and cross-spectral characteristics are analyzed. On the base of such analysis the stochastic model of the system response is grounded and the approximation expressions for the description of its covariance and spectral structure are proposed. The opportunity of usage of the proposed quantities for the crack detection is shown. The result of the influence of the damping coefficient on the value of indicators, which are proposed to use for the crack detection, is given too. The principled difference between dependencies of these quantities with those, which are the proper to indicators formed on the base of amplitudes of the harmonics of the deterministic part, is established.

It should be noticed, that response realizations analyzed in the given paper are obtained on the base of simulation of the bilinear single-degree-of-freedom (BSDOF) model [1–4]. The results of FFT spectra analysis of BSDOF system response quantitatively differ from these, which are obtained on the base of 2-D FEM [11–14], hence, as it is underlined in [13], BSDOF model "can be useful firstly to explore a wide range of problem parameters with low computational efforts".

The defined properties of forced oscillations for excitation used in this paper show that their periodic nonstationarity of the second order is a result of system nonlinearity, which appears after crack initiation. The quantities, which characterize such nonstationarity, i.e. covariance and spectral components, are quite sensitive to crack size. The vibration signals in real life are significantly different from harmonic and are characterized by the presence of deterministic as well as stochastic parts. Therefore, the diagnostic indicators formed on the base of covariance or spectral components are very useful for rotary machinery monitoring under operational conditions. The paper results confirm reasonability of laboratory tests of beam elements with using of excitations in the form of a sum of harmonic and stochastic oscillations.

Synopsis: the paper consists of Introduction, three sections divided into subsections and Conclusions. The properties of nonlinear oscillator model, which is investigated, and the applied force are described in Sections 2.1 and 2.2. The basic properties of mean, covariance function and spectral density of PCRP – model of forced oscillations – are given in Section 3.1. The methods of these characteristics estimation, which are used for simulated realizations processing, are considered in Section 3.2. The properties of the estimators of mean function and variance are calculated for various crack lengths and analyzed in Section 4.1. Comparison of the damage indicators formed on the base of amplitudes of the mean function harmonics and variance harmonics – covariance components – is carried out in Section 4.2. Section 4.3 is dedicated to analysis of the changes of response covariance structure as crack grows, to ground of mathematical model of forced oscillations and to approximation of covariance structure of PCRP, which describes oscillation's stochastic part. Spectral properties of stochastic modulations of the response the first and the second harmonics, which appears after crack initiation, and the opportunity of the spectral characteristics applying for crack detection are analyzed in Section 4.4. The investigation of dependencies of deterministic damage indicator as well as stochastic one on damping coefficient is provided in Section 4.4.

2. BSDOF model of cracked beam

2.1. The oscillator nonlinear properties

It was indicated that we will analyze cracked beam oscillations on the basis of numerical solutions of the following nonlinear differential equation of the second order [2,4]:

$$\frac{d^2\xi}{dt^2} + 2\beta\frac{d\xi}{dt} + \omega_c^2\left[1 - \frac{\alpha}{2}(1 + \mathrm{sgn}\xi)\right]\xi = \eta(t) \tag{1}$$