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Uncertainty propagation for nonlinear vibrations: A non-intrusive approach ☆

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ABSTRACT

The propagation of uncertain input parameters in a linear dynamic analysis is reasonably well established today, but with the focus of the dynamic analysis shifting towards nonlinear systems, new approaches is required to compute the uncertain nonlinear responses.

A combination of stochastic methods (Polynomial Chaos Expansion, PCE) with an Asymptotic Numerical Method (ANM) for the solution of the nonlinear dynamic systems is presented to predict the propagation of random input uncertainties and assess their influence on the nonlinear vibrational behaviour of a system. The proposed method allows the computation of stochastic resonance frequencies and peak amplitudes based on multiple input uncertainties, leading to a series of uncertain nonlinear dynamic responses. One of the main challenges when using the PCE is thereby the Gibbs phenomenon, which can heavily impact the resulting stochastic nonlinear response by introducing spurious oscillations. A novel technique to avoid the Gibbs phenomenon is be presented in this paper, leading to high quality frequency response predictions.

A comparison of the proposed stochastic nonlinear analysis technique to traditional Monte Carlo simulations, demonstrates comparable accuracy at a significantly reduced computational cost, thereby validating the proposed approach.

1. Introduction

Mechanical systems can experience nonlinear dynamic behaviour such as cubic stiffness, contact, friction, or impact, which can have a significant influence on the dynamic response of the system. In this case a conventional linear analysis becomes insufficient to describe the behaviour of the physical system and a nonlinear dynamic analysis is required. A large research effort has focused on the prediction of such nonlinear responses, where most techniques are based on numerical integration over time [1], providing accurate but computationally expensive results.

A more efficient technique to obtain the nonlinear dynamic response is the combination of the Harmonic Balance method (HBM) with a continuation predictor–corrector method [2] allowing the computation in the frequency domain. To further improve the computational efficiency of the approach, the iterations of the correction steps can be replaced by the

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Asymptotic Numerical Method (ANM) [3,4]. This method is based on a high order predictor to avoid the expensive correction step. The major restriction of the ANM is that the nonlinearities must be expressed in a quadratic form, somewhat restraining its wide applicability, especially for complex industrial problems where the non-linearities cannot be accurately represented by a polynomial quadratic form. However, several kinds of nonlinearities have been successfully processed with the ANM [5–7] leading to a significant reduction in the computational time.

With the emergence of reliable and accurate prediction techniques for the nonlinear response, the influence of uncertainties on the vibration behaviour has become of major interest to allow accurate predictions over a wide range of initial conditions. Uncertainty quantification (UQ) has been used to predict the effects of possible uncertainties, originating from manufacturing processes or operational conditions of a real structure, and to derive models that can take them into account. Traditional Monte Carlo simulations (MCS) [8] are often used for this purpose. They provide accurate results, but have very slow convergence rates, which limits their use for nonlinear dynamic analysis somewhat. To overcome some of the issues with the MCS, other stochastic methods have been suggested, including the Polynomial Chaos Expansion (PCE), introduced by Wiener [9] who represented a stochastic process using a series of Hermite polynomials with Gaussian random variables. The PCE has been successfully applied to model uncertainties in linear finite elements applications [10], where the uncertain Gaussian input parameters were expressed via the Karhunen–Lòeve Expansion and the system response was determined using the PCE.This technique has been generalised to non-Gaussian random variables [11,12] using orthogonal polynomial basis adapted to any probability distribution function. In further generalisations, the PCE has been extended to rational function series [13], partitioned random space [14] and sparse chaos expansions [15], adding additional features and enabling its use for the presented nonlinear dynamic analysis with uncertainties.

The PCE has been used, in combination with the HBM, to analyse the dynamic linear response of a rotor with Gaussian uncertainties [16], saving significant computational time when compared to MCS without the loss of accuracy. For this linear case the frequency was considered to be independent of the uncertain variables which is unsuitable for a nonlinear dynamic response analysis. The latter can be characterised by returning points (and so multi-solutions) which makes the frequency non-deterministic. To overcome this issue, Didier and Sinou [17] used an intrusive PCE in combination with the HBM and a predictor–corrector method, leading to an accurate stochastic response of a nonlinear system. This approach has been successfully applied to complex mechanical problems [18], but unfortunately the intrusive nature requires a rewrite of the equations of the problem which in turn necessitates modifications to the deterministic numerical solver for each new computation. The intrusive approach also requires the definition of a stochastic phase condition which differs for each type of nonlinearity and which is usually quite difficult to find.

A large number of PCE coefficients are often required to achieve convergence of the first and second statistical moments (mean and standard deviation) to the MCS solution [19], reducing somewhat the computational advantages of PCE. To improve the efficiency of the PCE, a combination with Aitken's method has been proposed in [20] leading to an improvement of the convergence rate.

A further problem arising from the use of PCE is the presence of spurious oscillations in the stochastic response in the case of strong discontinuities. This so called Gibbs phenomenon can strongly affect the results and introduce large differences to the reference solution (eg. MCS) [17,21]. This is of particular relevance for nonlinear dynamic problems, where the presence of returning points in the response can lead to significant oscillations, making an accurate and reliable prediction quite challenging.

In this paper an efficient approach to compute the stochastic nonlinear dynamic response of a system will be presented, combining the PCE with the HBM and the ANM to allow an accurate and fast computation. The introduced method can be applied as an intrusive or a non-intrusive approach, the latter giving it a much wider applicability for industrial application since it eliminates the need for modifications to existing deterministic solvers. The proposed technique also avoids the Gibbs oscillations, enabling an accurate and fast computation of the stochastic nonlinear dynamic response, which is further improved by the use of the Smolyak quadrature [22] for multivariate random spaces. The resulting frequency response functions are validated against Monte Carlo Simulations (MCS).

2. Deterministic nonlinear problem

The chosen approach to solve the deterministic nonlinear problem is a combination of the Harmonic Balance method (HBM, [4]) with the Asymptotic Numerical Method (ANM) that allows a fast and accurate solution of the nonlinear equations. The ANM is based on a Taylor expansion of the solution [3] and its combination with HBM was first introduced by Moussi [5]. A short introduction to the HBM+ANM approach will be provided for completeness, but for a detailed discussion the reader may consult above references.

ANM is a continuation method based on the expression of the solution in a Taylor series. The initial solution (first point) is calculated using a traditional iterative process (e.g. Newton's method) after which the solution path is followed by approximating the solution with a Taylor series. When enough terms are being used in the Taylor series, the error on the computed solution is very small, eliminating the need for the time consuming correction step of traditional path following techniques.

One of the formulations of a nonlinear dynamic system characterised by D DOFs is: