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Information transport in classical statistical systems

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Abstract

For "static memory materials" the bulk properties depend on boundary conditions. Such materials can be realized by classical statistical systems which admit no unique equilibrium state. We describe the propagation of information from the boundary to the bulk by classical wave functions. The dependence of wave functions on the location of hypersurfaces in the bulk is governed by a linear evolution equation that can be viewed as a generalized Schrödinger equation. Classical wave functions obey the superposition principle, with local probabilities realized as bilinears of wave functions. For static memory materials the evolution within a subsector is unitary, as characteristic for the time evolution in quantum mechanics. The space-dependence in static memory materials can be used as an analogue representation of the time evolution in quantum mechanics – such materials are "quantum simulators". For example, an asymmetric Ising model on a Euclidean two-dimensional lattice represents the time evolution of free relativistic fermions in two-dimensional Minkowski space.

1. Introduction

Memory and information transport are key issues in information technology. The study of statistical systems of Ising spins or bits has shaped the conceptual advances for the role of information [1]. The understanding of computations with materials able to conserve memory [2–5] may well influence future information processing. In this paper we propose a formalism for the problem of (static) information transport based on the concept of classical wave functions. It resembles the derivation of the wave function from the path integral in quantum mechanics by

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Feynman [6]. However, our approach remains entirely rooted in classical statistics, describing classical Ising spins in thermal equilibrium or more elaborate static states in classical statistical systems.

We investigate probability distributions in classical statistics for systems with boundaries and a "bulk" limited by the boundaries. How is a change in the boundary conditions reflected by the observables within the bulk? This amounts to the question how a signal propagates from the boundary into regions within the bulk or how information is transported within the bulk. In turn, this issue is directly related to the question how information can be transported from one boundary to another, say between the two ends of a wire. We address these questions in a static context, for example thermal equilibrium, without any dependence on time. The absence of a genuine time evolution associates the bulk of such static states to a generalized notion of "equilibrium state", even for situations where the latter is not unique.

One finds a rather rich variety of different possible behaviors for the information transport. For the most common situation the boundary properties are not relevant for the bulk. Boundary information is lost within a finite correlation length, either monotonically or as damped oscillations. This situation is realized if the bulk equilibrium state is unique. A neighboring case is the power-like decay in case of an infinite correlation length, as for critical phenomena. Most interesting in our context are static memory materials for which expectation values of observables in the bulk depend on the boundary conditions. This becomes possible if no unique bulk equilibrium state exists, as in case of spontaneous symmetry breaking. Information can now be transported from the boundary to the bulk, imprinted on the properties of the degenerate "equilibrium states." For example, we find models with oscillating local probabilities in the bulk, with details of the oscillations depending on the boundary conditions.

In general, local probabilities and therefore the expectation values of local observables $A(t)$ will depend on the location t of some hypersurface in the bulk. For an Ising model with a finite correlation length ξ the boundary information will be exponentially erased for $\Delta t = \min(t_f - t, t - t_{in})$ larger than ξ

$$\langle A(t)\rangle = \bar{A} + c_A \exp(-\Delta t/\xi), \tag{1}$$

with $\bar{A}$ the "bulk expectation value" or equilibrium value. At a phase transition ξ diverges and $\langle A\rangle$ is approached with a power law in Δt. We will see that this loss of memory of boundary conditions is characteristic for all systems with a unique equilibrium state.

The loss of memory of boundary information is, however, not the only possibility. One may ask under which circumstances memory of boundary conditions is kept, for example by an oscillating behavior as

$$\langle A(t)\rangle = a_0 \cos(\omega t + \alpha), \tag{2}$$

with α depending on the boundary conditions. We find such a behavior for highly interesting "static memory materials" where bulk observables keep high sensitivity to boundary conditions. Static memory materials can be realized if no unique equilibrium state for the bulk exists. Degenerate generalized equilibrium states occur, in particular, in case of spontaneous symmetry breaking or in presence of conserved quantities.

A simple example for a static memory material is the asymmetric diagonal two-dimensional Ising model, with action

$$S_{cl} = -\frac{\beta}{2}\sum_{t,x}\big[s(t+\epsilon, x+\epsilon)\, s(t,x) - 1\big], \tag{3}$$