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Nonlinear Vibrations of a Cable System with a Tuned Mass Damper under Deterministic and Stochastic Base Excitation

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Abstract

This paper investigates a dynamic model of a cable – mass system equipped with an auxiliary mass element to act as a transverse tuned mass damper (TMD). The cable length varies slowly while the system is mounted in a vertical host structure swaying at low frequencies. This results in base excitation acting upon the cable - mass system. The model takes into account the fact that the longitudinal elastic stretching of the cable is coupled with their transverse motions. The TMD is applied to reduce the dynamic response of the system. The parameters of TMD are selected by the application of a linearized model and a single-mode approximation. In this approach the excitation is represented as a narrow-band Gaussian process mean-square equivalent to a harmonic process. The deterministic model and stochastic model are used to predict and control the resonance response of the system.

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Keywords: Cable System; Slowly Varying; Vibrations; Tuned Mass Damper

1. Introduction

Moving cable systems are deployed in many engineering systems. In some applications the length of cables vary during operation rendering the system non-stationary. For example, in hoist, elevator and mine lifting installations the payload- carrying cables moving at speed within a host structure have time-variant length and the natural frequencies vary with their length [1]. The modular cable - mass installations are mounted within host structures that

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are often subject to environmental phenomena such as wind and seismic excitations [2]. The corresponding response and excitation mechanisms can be represented by deterministic functions or treated as stochastic processes [3,4]. In this paper a deterministic model and the corresponding stochastic model of a mass –cable system constrained to move vertically in a host structure are considered. The system is equipped with an auxiliary spring – damper - mass combination attached to the main (primary) mass to act as a *tuned mass damper* (TMD). In this arrangement the TMD can be applied to mitigate the effects of resonance when the frequency of the base motion becomes near the natural frequency corresponding to the primary mass – cable mode.

2. Mathematical model

2.1. System Configuration

Fig. 1 shows a mass – vertical cable system mounted within a host structure with a primary mass *M* attached to the lower end of the cable of time-varying length $L = L(t)$ moving axially at transport speed *V*. The cable is mounted within a host structure of height $AB = Z_0$ with its upper end passing thrpough *O* at the top of the structure. The mean quasi-static tension, mass per unit length, modulus of elasticity and cross-sectional metallic area of the cable are denoted as $T^i = \left[M + m_d + m(L-x)\right](g-a)$, *m, E* and *A*, respectively. The Eulerian spatial coordinate *x* is measured from the upper end downwards as shown. The lateral dynamic displacements of the cable are denoted as *v(x,t)*. They are coupled with the longitudinal vibrations denoted as *u(x,t)*. The mass *M* is constrained in the lateral direction by a linear spring of coefficient of stiffness *k* and can move in the vertical direction. Its lateral and longitudinal vibrations are denoted as $v_M(t)$ and $u_M(t)$, respectively. An auxilliary small mass m_d is attached to the main mass via a spring – dashpot system of coefficient of stiffness k_d and coefficient of viscous damping c_d, respectively. The auxilliary mass is constrained to mover horizontally with its motion denoted as z_d. The equations of motion Eq. (1) are developed by applying the extended Hamilton's principle.

$$
\begin{aligned}
& m\frac{D^2u}{Dt^2} - EA\varepsilon_x = 0,\ \ m\frac{D^2v}{Dt^2} - Tv_{xx} + m(g-a)(xv_{xx} + v_x) - EA(\varepsilon v_x)_x = 0 \\
& M\ddot{v}_M + T^i(L)v_x\big|_{x=L} + k\Delta - k_d(z_d - v_M) - c_d(\dot{z}_d - \dot{v}_M) + EA\,\varepsilon\big|_{x=L} v_x\big|_{x=L} = 0 \\
& m_d\ddot{z}_d + k_d(z_d - v_M) + c_d(\dot{z}_d - \dot{v}_M) = 0,\ \ (M + m_d)\ddot{u}_M + EA\,\varepsilon\big|_{x=L} = 0
\end{aligned} \tag{1}
$$

where $\varepsilon = u_x + v_x^2/2$ represents the axial strain, $D(\ )/Dt = (\ )_t + V(\ )_x$ $(\ )_t$ and $(\ )_x$ represent partial derivatives with respect to time *t* and *x*, respectively, and $T = (M + m_d + mL)(g-a)$, where *a* represents the acceleration of the transport motion. For tensioned members such as metallic cables the lateral frequencies are much lower than the longitudinal frequencies. Thus, considering that the excitations frequencies are much lower than the fundamental longitudinal frequencies the longitudinal inertia of the cable can be neglected in the first equation in (1). Thus, this equation can be integrated to give $u_x = e(t) - v_x^2/2$ where *e(t)* represents the quasi-static axial strain in the cable.

2.2. Base Excitation

The host structure is subjected to bending deformations acting as base excitation and described by the polynomial shape function $\Psi(\eta) = 3\eta^2 - 2\eta^3$ (see Fig. 1), where $\eta = z/Z_0$ with *z* denoting a coordinate measured from ground level and Z_0 representing the height at the top end of the cable. In this scenario the structure undergoes harmonic motions $v_0(t)$ of frequency Ω_0 and amplitude A_0, measured at the level Z_0. Thus, at the upper end the displacements of the cable are $v(0,t) = v_0(t)$. In order to accommodate the base excitation in the equations of motion (1) the overall lateral displacements of the cable – mass system are expressed by Eq. (2).

$$
v(x,t) = \overline{v}(x,t) + \left(1 + \frac{\Psi_L - 1}{L}x\right)v_0(t),\ \ \Psi_L = \Psi\left(\frac{Z_0 - L}{Z_0}\right) \tag{2}
$$