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Probabilistic load flow calculation by using probability density evolution method

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ABSTRACT

This paper presents a novel framework methodology based on the probability density evolution method (PDEM) for solving the probabilistic load flow (PLF) problem. By leveraging a constructed visual stochastic process, the joint probability density evolution equation of a system statement and random inputs is derived based on the principle of preservation of probability. The probability density function of the system statement can then be numerically solved by means of a TVD-based difference scheme. The proposed method is validated through case studies in which the active power and reactive power consumptions of buses are assumed to obey a normal distribution and Weibull distribution, respectively. The cumulative probability functions of the voltage magnitudes of buses and active power branches are computed using the PDEM with 100 samples. The mean, standard deviation, skewness, and kurtosis are also examined. The comparison to Monte Carlo of 10,000 simulations demonstrates the accuracy and efficiency of the proposed approach and verifies its suitability to solve the PLF problem.

1. Introduction

Load flow calculation is a fundamental issue for state estimation and system planning in the electric power industry. In addition to random input loads, the integration of wind energy and photovoltaic power has introduced uncertainty into electric network systems. Therefore, probabilistic load flow (PLF) analysis must be utilized to handle variation in state variables, such as bus voltages and line flows. Historically, load flow analysis considering randomness was first proposed by Borkowska [1] in 1974. Many mathematical methods handling uncertainty have since been introduced to the PLF problem, including fuzzy theory [2,3], set theory[4], interval methods [5], and probability theory. Additionally, unscented transformation [6], Gaussian mixture models [7], and univariate dimension reduction methods [8] have also been adopted to perform PLF analysis.

Among these methods, probability theory-based methods are the most popular. Initially, the convolution technique [9] dominated PLF analysis by linearizing load flow equations. Thereafter, owing to the essential limitations of linearization and rapid development of computational capacity, the convolution technique was replaced by other methods that are capable of handling nonlinearity in load flow equations. Among these methods, Monte Carlo simulation [10,11] is particularly notable for its generality. Because the Monte Carlo method requires a large number of simulations to ensure accuracy, it is often utilized to verify the accuracy of other methods instead of being used directly in engineering practice. To reduce the time cost of Monte Carlo simulation with simple or direct sampling, various modified sampling strategies (e.g., importance sampling, stratified sampling, and quasi-Monte Carlo methods [11–13]) have been proposed. Additionally, other approximate methods have also been developed to balance calculation efficiency and accuracy. Combinations of cumulants and Gram-Charlier expansions [14] or Cornish-Fisher expansions [15] have been employed to calculate the PLF. Furthermore, a series of point estimation methods [16–18] have also been utilized to analyze PLF. Recently, considering the essential randomness of photovoltaic systems and using an index of photovoltaic penetrations, Ruiz-Rodriguez et al. [19] investigated the impact of the size of a single-phase photovoltaic system on voltage unbalance in a secondary radial distribution network using a point estimation method. Hernández et al. [20] assessed the impact of uncertainty in electric vehicles and photovoltaic generation on radial distribution systems. They proposed a general analytical technique based on the Cornish-Fisher expansion and a finite mixture distribution to handle the non-stationarity of loads. It is noteworthy that in the absence of a probabilistic distribution of random input variables, the point estimation method is a more attractive option.

In recent years, based on the principle of preservation of

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probability, the probability density evolution method (PDEM) was been developed by Li and Chen [21,22]. Initially, it was adopted to predict structural responses in the presence of random excitations [23] or uncertain parameters [24]. Thus far, the PDEM has been successfully applied to seismic response and reliability analysis [25], static random buckling analysis [26], dynamic buckling analysis incorporating structural imperfections [27], fatigue reliability evaluation [28], and several other fields [29]. This paper proposes a novel methodology based on the PDEM to compute PLF.

The remainder of this paper is organized as follows. The problem statement and formulas for PLF based on the PDEM and a numerical scheme are introduced in Section 2. Next, case studies with different coefficients of variation for normal inputs and Weibull inputs are presented to validate the proposed method in Section 3. Finally, various conclusions are drawn in Section 4.

2. Problem statement

2.1. Formulation

This section presents a general framework for the PLF problem based on PDEM theory. Based on the formal solution for load flow analysis from probability density evolution theory, the joint probability density evolution equations of a system statement and random inputs are derived. Power flow analysis is performed to determine the steady operating state of a power system. Load demands and power generation are typically given, and the voltages (magnitudes and angles) of buses and the active and inactive power through lines can be obtained by solving the governing equations of the power system. The nonlinear load flow equations [13,30] for a power system can be expressed as:

$$\begin{aligned}\mathbf{y} &= g(\mathbf{x})\\ \mathbf{z} &= h(\mathbf{x})\end{aligned} \tag{1}$$

where $\mathbf{x}$ denotes the state vector of nodal voltages and angles, $\mathbf{y}$ represents the vector of real and reactive power injections, $\mathbf{z}$ is the vector of real and reactive line flows, and $g(\cdot)$ and $h(\cdot)$ are nonlinear power injection functions and line flow functions, respectively.

When load demand is uncertain, both the input and output of the power system need to be treated as random variables. Let Y be the state variable of interest. Then, its formal solution can be written as:

$$Y(\Theta) = G(\mathbf{X},\Theta) \tag{2}$$

where $\mathbf{X}$ denotes the set of determine input parameters (e.g., system admittance), Θ is the set of random parameters (e.g., active or inactive injections of buses), and G is a transformation from considered factors to the statement variables of interest based on Eq. (1), which typically needs to be solved using numerical methods [31].

Because load flow calculation is essentially modeled as a steady-state problem, in order to employ the time-dependent PDEM to perform PLF analysis, a virtual stochastic process [25] must first be constructed as follows:

$$\Phi(\Theta,\tau) = Y(\Theta)\cdot\tau = G(\mathbf{X},\Theta)\cdot\tau \tag{3}$$

Here, a virtual time τ is introduced to form a stochastic process $\Phi(\Theta,\tau)$. One can see that the random variable of interest can be obtained using $Y(\Theta) = \Phi(\Theta,\tau = 1)$. Additionally, the deviation of Φ with respect to τ can be obtained as follows:

$$\dot{\Phi}(\tau) = \frac{\partial\Phi}{\partial\tau} = Y(\Theta) \tag{4}$$

For time-dependent loads, there is no need to construct the virtual stochastic process. One can directly solve the transient governing equations to obtain the probability density function (PDF) of interest by using the PDEM.

With the help of the stochastic process $\Phi(\Theta,\tau)$, the PDF $p_Y(y)$ of Y can be determined by using the PDEM. First, the joint probability density evolution equation $p_{\Phi\Theta}(\phi,\theta,\tau)$ of $\Phi(\Theta,\tau)$ and random parameters Θ are derived based on the principle of preservation of probability. Consequently, the PDF of Y can be obtained by integrating the joint PDF $p_{\Phi\Theta}(\phi,\theta,\tau)$ at $\tau = 1$ over the defined domain of the random parameters Θ. In other words, one can obtain the PDF of Y by using the following relationship:

$$p_Y(y) = \int_\Theta p_{\Phi\Theta}(\phi,\theta,\tau = 1)d\theta \tag{5}$$

The strategy for the derivation of the joint PDF $p_{\Phi\Theta}(\phi,\theta,\tau)$ is formally similar to the conservation equations in computational fluid dynamics. In this study, the statement variable of a power system is modelled as a virtual stochastic process, where its actual statement corresponds to a particular instant with the virtual time being equal to 1. Because there are neither additional new random factors nor disappearing random factors in the virtual evolution process under consideration, $\{\Phi(\Theta,\tau),\Theta\}$ defined at $\{\Omega_\Phi \times \Omega_\Theta\}$ is a preserved stochastic system, where Ω_Φ is the defined domain of the virtual stochastic process and Ω_Θ is the defined domain of the random parameters under consideration.

Now, let Ω be an arbitrary subdomain in $\{\Omega_\Phi \times \Omega_\Theta\}$, where $\partial\Omega$ is the boundary of Ω. We inspect the change in probability in Ω and the probability flowing into Ω through $\partial\Omega$ during the virtual time interval $d\tau$. First, the probability increment through $\partial\Omega$ during $d\tau$ is denoted:

$$-(p_{\Phi\Theta}\dot{\phi}d\tau)\cdot\mathbf{n}dS_\Phi d\theta \tag{6}$$

where $\dot{\phi}$ is the deviation of ϕ with respect to τ. Let $\partial\Omega_\Phi$ be the boundary of Ω_Φ, where S_Φ denotes the area element of $\partial\Omega_\Phi$, $\mathbf{n}$ is the norm vector of $\partial\Omega_\Phi$, and $d\theta$ denotes the volume element of Ω_Θ. Additionally, the minus symbol indicates that the probability flows into Ω.

Next, the joint PDF $p_{\Phi\Theta}(\phi,\theta,\tau)$ at $\tau + d\tau$ can be expressed by first order expansion as:

$$p_{\Phi\Theta}(\phi,\theta,\tau + d\tau) = p_{\Phi\Theta}(\phi,\theta,\tau) + \frac{\partial p_{\Phi\Theta}(\phi,\theta,\tau)}{\partial\tau}d\tau \tag{7}$$

Therefore, the incremental probability in Ω during $d\tau$ is:

$$\frac{\partial p_{\Phi\Theta}(\phi,\theta,\tau)}{\partial\tau}d\phi d\theta d\tau \tag{8}$$

Because no new random factors are considered, according to the principle of preservation of probability [19,29], the incremental probability in Ω must be equal to the probability flowing into Ω through $\partial\Omega$:

$$\int_{\Omega_\Phi\times\Omega_\Theta}\frac{\partial p_{\Phi\Theta}(\phi,\theta,\tau)}{\partial\tau}d\phi d\theta d\tau = -\int_{\partial\Omega_\Phi\times\Omega_\Theta}(p_{\Phi\Theta}\dot{\phi}d\tau)\cdot\mathbf{n}dS_\Phi d\theta \tag{9}$$

By using the Gaussian integration theorem, the right side of Eq. (9) can be converted into an integration over $\{\Omega_\Phi \times \Omega_\Theta\}$ and we have:

$$-\int_{\partial\Omega_\Phi\times\Omega_\Theta}(p_{\Phi\Theta}\dot{\phi}d\tau)\cdot\mathbf{n}dS_\Phi d\theta = -\int_{\Omega_\Phi\times\Omega_\Theta}\frac{\partial(p_{\Phi\Theta}Y(\Theta))}{\partial\phi}d\phi d\theta d\tau \tag{10}$$

By substituting Eq. (10) into Eq. (9) and noting that $Y(\Theta)$ is independent over ϕ, one obtains:

$$\int_{\Omega_D\times\Omega_\Theta}\left\{\frac{\partial p_{\Phi\Theta}(\phi,\theta,\tau)}{\partial\tau} + Y(\Theta)\frac{\partial p_{\Phi\Theta}(\phi,\theta,\tau)}{\partial\phi}\right\}d\phi d\theta d\tau = 0 \tag{11}$$

Considering the arbitrary nature of Ω, the joint probability density evolution equation $p_{\Phi\Theta}(\phi,\theta,\tau)$ is derived as:

$$\frac{\partial p_{\Phi\Theta}(\phi,\theta,\tau)}{\partial\tau} + Y(\Theta)\frac{\partial p_{\Phi\Theta}(\phi,\theta,\tau)}{\partial\phi} = 0 \tag{12}$$

The corresponding initial condition can be written as:

$$p_{\Phi\Theta}(\phi,\theta,\tau)|_{\tau=0} = \delta(\phi)p_\Theta(\theta) \tag{13}$$

where $\delta(\phi)$ is Dirac's delta function. The initial condition implies that the probability distribution of random parameters was determined at the initial state.